Math Objectives

- Students will be able to identify the conditions that determine when nested triangles that share a common angle are similar triangles.
- Students will validate conditional statements about nested triangles that share a common angle.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

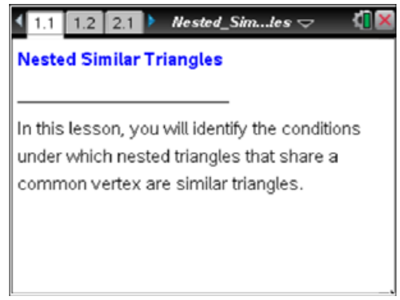
- similar polygons
- corresponding parts
- parallel
- nested triangles

About the Lesson

- In this lesson, nested triangles have been created so that they share a common vertex and vertex angle.
- The students will drag the endpoints of the side that is opposite the common angle to discover the conditions that make the nested triangles similar. When students make congruent angle marks appear, the triangles will be similar.
- In one case the sides opposite the common angle will be parallel; in another case they will not.
- As a result students will:
  - Observe that the ratios of the corresponding sides are equal.
  - Formulate and evaluate conditional statements that deal with the similarity of nested triangles.

TI-Nspire™ Navigator™ System

- Use Quick Poll to check student understanding.
- Use Screen Capture to examine patterns that emerge.
- Use Live Presenter to engage and focus students.
- Use Teacher Edition computer software to review student documents.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
Nested_Similar_Triangles_Student.pdf
Nested_Similar_Triangles_Student.doc

TI-Nspire document
Nested_Similar_Triangles.tns

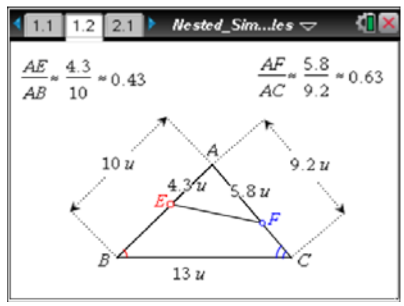
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (✋) getting ready to grab the point, not a hand pointing at the point (👆). Press ctrl 🖐 to grab the point and close the hand (✊).

Tech Tip: Students need to drag *E* and *F* slowly so that they will see the congruent angle marks appear in the nested triangles. If the hand stays on the point, it is easier to get the angle marks to show. Once the angle marks show, immediately pressing esc will ensure that the angle marks stay.

Move to page 1.2.



1. Drag point *E* to any position on $\overline{AB}$. Then slowly drag point *F* until congruent angle marks appear for $\measuredangle AEF$ and $\measuredangle B$, and press enter.
   a. Explain why $\measuredangle AFE$ and $\measuredangle C$ are also congruent.

Answer: The measure of $\triangle AFE$ can be found by subtracting the measures of $\measuredangle A$ and $\measuredangle AEF$ from 180°. The measure of $\measuredangle C$ can be found in a similar way. Since $\measuredangle A$ is the common angle and $\measuredangle AEF$ and $\measuredangle B$ have the same measures, this difference will be equal.

   b. When these pairs of angles are congruent, what is the relationship between the two ratios?

Answer: The ratios $\frac{AE}{AB}$ and $\frac{AF}{AC}$ are equal. For example, they could both equal 0.44.

Teacher Tip: Numerical answers will vary. Later students will drag *E* and *F* to other positions that also produce congruent angles. At this point you can ask students to compare their screens to see if there are different numerical values for the ratios.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

c. Write a proportion using $\overline{AE}$, $\overline{AB}$, $\overline{AF}$, and $\overline{AC}$.

Sample Answer: Answers may vary; for example: $\frac{AE}{AB} = \frac{AF}{AC}$.

2. When $\measuredangle AEF$ is congruent to $\measuredangle B$, is $\overline{EF}$ parallel to $\overline{BC}$? Why or why not?

Answer: If $\measuredangle AEF$ and $\measuredangle B$ are congruent, then $\overline{EF}$ is parallel to $\overline{BC}$ because these two angles are corresponding angles formed by two lines cut by transversal $\overline{AB}$.

> **Teacher Tip:** If students answered 1a by using parallel lines, this question may seem redundant. Make sure students who use the triangle sum of 180° to answer 1a see that the sides opposite the common angle are parallel.

3. The nested triangles are similar.
   a. Finish the similarity statement for these triangles. (Note: the order of the vertices matters.)

Answer: $\Delta ABC \sim \Delta AEF$

b. What evidence shows that the two triangles are similar?

Answer: All the pairs of corresponding angles are congruent.

> **Teacher Tip:** Some students may answer that the corresponding sides formed on the sides of the common angle are proportional. Thus, the triangles are similar by the SAS similarity theorem. Point this out to the class if they answer with the more obvious AA similarity theorem.

c. How does $\overline{EF} : \overline{BC}$ compare to the other ratios found in the similar triangles? Justify your answer.

Answer: The ratio will be equal to the other ratios because all corresponding sides in similar triangles are proportional.

> **Teacher Tip:** Students can use the **Measurement** tool to measure the length of $\overline{EF}$ and verify that $\overline{EF} : \overline{BC}$ is equal to the other ratios.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

4. Drag point *E* to a new position on $\overline{AB}$ and slowly drag point *F* until the congruent angle marks appear. Do any of your answers to questions 1–3 change? Explain.

Answer: The length of segments $\overline{AE}$, $\overline{AF}$, and $\overline{EF}$ changed, so the values of the ratios changed, but the other answers would not change. With the congruent angles, $\overline{EF}$ will still be parallel to $\overline{BC}$, the sides will be proportional, and the triangles will still be similar.

5. a. If two triangles share a common angle, are they always similar? Why or why not?

Answer: No. When students drag point *F*, the only time the triangles are similar is when the congruent marks appear. When $\overline{EF}$ and $\overline{BC}$ are not parallel, the triangles are not necessarily similar.

b. If you have nested triangles with $\measuredangle A$ in common, what conditions are necessary for the triangles to be similar? Write the statement(s) in if-then form.

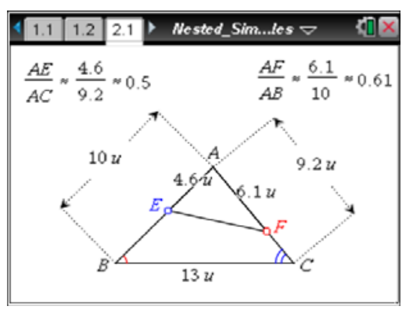
Sample Answers:

(1) If you have nested triangles with $\measuredangle A$ in common and at least 1 other angle in 1 triangle is congruent to the corresponding angle in the other triangle, then the nested triangles are similar.

(2) If you have nested triangles with $\measuredangle A$ in common and the sides opposite $\measuredangle A$ are parallel, then the nested triangles are similar.

(3) If you have nested triangles with common $\measuredangle A$ and the corresponding sides that form $\measuredangle A$ are proportional, then the nested triangles are similar.

Move to page 2.1.



6. Drag the open circle at point *F* to any position on $\overline{AC}$. Then slowly drag the open circle at point *E* until congruent angle marks show. Then press **enter**.

a. When the congruence marks appear for angles, what do you observe about the ratios of the sides?

Answer: They are equal.

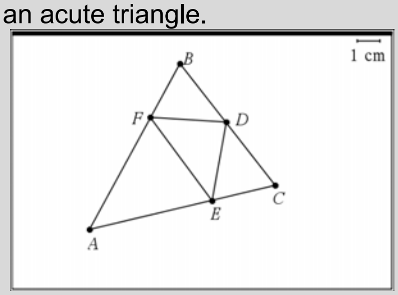
b. When the congruent marks appear, are the triangles similar? Why or why not?

Answer: The triangles are similar because all of the corresponding angles in the triangles are congruent.

c. If the triangles are similar, then finish the similarity statement for these triangles (Note: the order of the vertices matters.)

Answer: $\Delta ABC \sim \Delta AFE$

Teacher Tip: Nested triangles in which the corresponding sides and angles "twist" around the common angle of A are not obvious because the sides opposite the common angle are not parallel. There is a word for the relationship of these sides: they are said to be **antiparallel.** An example of these "twisted" similar nested triangles occurs when you connect the feet of the three altitudes in an acute triangle.



The triangle formed by the feet of the altitudes in an acute triangle is called the **orthic triangle.** It cuts the original triangle into four triangles. The three triangles at the vertices are similar to the original triangle.
$\Delta ABC \sim \Delta AEF \sim \Delta DBF \sim \Delta DEC$
The sides of the orthic triangle are antiparallels of the sides of the original triangle.

7. Consider the statements below.

   a. Ann said, “If two nested triangles are similar, then the sides opposite the common angle must be parallel.” Is she right? Explain.

   Answer: Ann is wrong. In the first problem, the sides were parallel. In the second problem, they were not parallel. The sides opposite the common angle do not have to be parallel.

   b. Kyle said, “If you can use $\overline{AE}$, $\overline{AB}$, $\overline{AF}$, and $\overline{AC}$ to make a proportion, then the nested triangles with $\measuredangle A$ in common are always similar.” Is he right? Explain.

   Answer: Kyle is right. In both problems, the triangles were similar, and we could put $\overline{AE}$, $\overline{AB}$, $\overline{AF}$, and $\overline{AC}$ in a proportion.

> **Teacher Tip:** Students may not put the proportions in the answer above. Make sure you talk about the proportions and how they are different.
>
> The proportion $\frac{AE}{AB} = \frac{AF}{AC}$ was used for question 1 and the proportion $\frac{AF}{AB} = \frac{AE}{AC}$ was used for question 2.

TI-Nspire Navigator Opportunity: *Quick Polls*
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- Nested triangles with a common angle are similar if and only if the two other angles in the smaller triangle are congruent to the two other angles in the larger triangle.
- Nested triangles with a common angle are similar if and only if the sides of the smaller and larger triangles that form the common angle can be used to write a proportion.
- If the sides opposite the common angle of nested triangles are parallel, then the nested triangles are similar. However, if they are not parallel, the nested triangles may still be similar.

Assessment

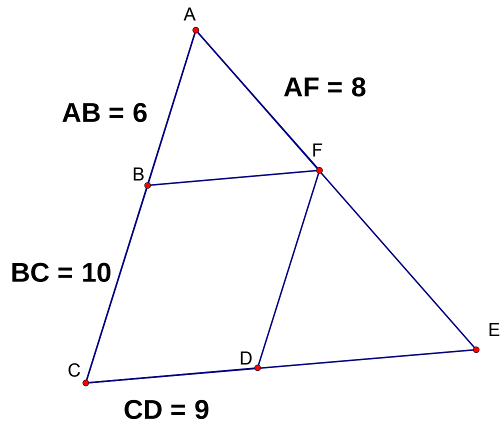
1. *BFDC* is a parallelogram. Find:

BF = ________

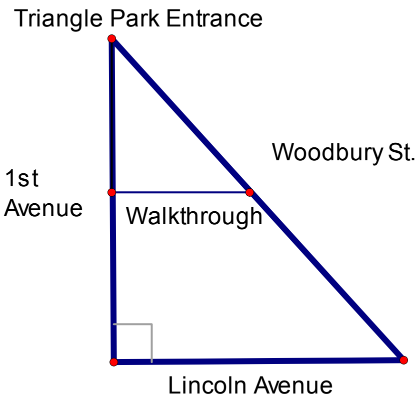
CE = ________

FE = ________

FD = ________



2.



The distance along 1st Avenue from the Triangle Park Entrance to the Walkthrough is 880 yards. The distance along 1st Avenue from the Triangle Park Entrance to Lincoln Avenue is 1,408 yards. The distance along Woodbury Street from the Walkthrough to Lincoln Avenue is 1,760 yards.

a. If the Walkthrough is parallel to Lincoln Avenue, find the distance from the Triangle Park Entrance to the Walkthrough along Woodbury.

b. If the Walkthrough is 495 yards long, what is the distance along Lincoln Avenue between 1st Avenue and Woodbury St?

Assessment answers:

1. BF = 9
   CE = 24
   FE = $\frac{40}{3}$
   FD = 10
2. a. $\frac{8800}{3}$ yards
   b. 792 yards

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Note 1

Question 1, Screen Capture: Use Screen Capture so that students can compare their screens to see if there are different numerical values for the ratios.

Note 2

Question 4, Live Presenter: Make a student the Live Presenter to measure $\overline{EF}$ and calculate $\overline{EF}:\overline{BC}$ for the whole class to verify that the ratio is equal to the other ratios.

Note 3

Question 7, Quick Polls (Multiple Choice or Open Response): Send students the following open response Quick Polls.

1. Using the similar nested triangles on page 1.2, if $AE = 4$, $AF = 6$, and $AB = 8$, what is AC?

   Answer: $AC = 12$

2. Using the similar nested triangles on page 2.1, if $AE = 4$, $AF = 6$, and $AB = 8$, what is AC?

   Answer: $AC = \frac{16}{3}$