

Independence Is The Word

ID: 11692

Time required

45 minutes

Activity Overview

In this activity, students will use a simulation to find the experimental probability of independent events. They will find the sample space and then compare the experimental and theoretical probabilities. Then students are introduced to dependent events and the formula for conditional probability.

Topic: Probability

- Simulations
- Independent and dependent events
- Conditional probability
- Law of large numbers
- Experimental and theoretical probabilities

Teacher Preparation and Notes

- This activity is intended to be an introduction to finding the theoretical probability for independent events. Specific directions and inquiry questions are part of the student worksheet so this can be an independent or group activity.
- Students will need groups to compile data.
- If time is limited, Problem 1 can be done in class the first day and Problem 2 and 3 can be done in class the second day.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter “11692” in the keyword search box.

Associated Materials

- Independence_Is_The_Word_Student.doc
- Independence_Is_The_Word.tns
- Independence_Is_The_Word_Soln.tns

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Simple Probability (TI-Navigator) — 1933
- Compound Events (TI-Nspire technology) — 10136
- Binomial Experiments (TI-Nspire technology) — 9999

Problem 1 – Simulating Independent Events

As an introduction, explain to students the definition of independent events and give the following examples listed.

Definition:

Two events A and B are **independent** if knowing the outcome of one does not change the probability that the other occurs.

Examples:

Independent: Each time a coin is tossed it is an independent event since the outcome of the first toss does not influence the outcome of the second toss.

Not Independent: Picking cards from a standard deck of cards is not independent since knowing the outcome of the first draw does change the probability for the second draw.

Review with students how to use the **Random Integer** generator to simulate the tossing of a coin. The command **randint(0, 1)** will generate one toss, where 0 represents heads and 1 represents tails. The command **randint(0, 1, 50)** will generate 50 tosses.

To start the activity, students are to predict the probabilities of different outcomes when tossing a coin twice.

In the simulation, students will let 0 represent heads and 1 represent tails. On page 1.4, in Columns A and B, students are to enter the command **=randint(0, 1, 100)**. Each row will represent one trial consisting of 2 coin tosses, for a total of 100 trials.

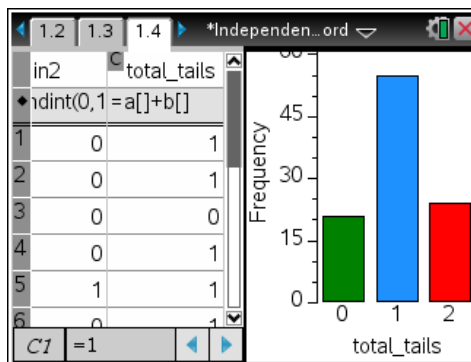
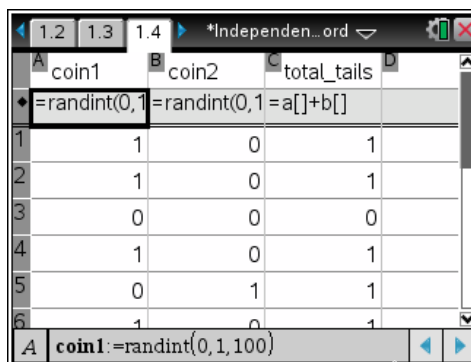
Then in Column C, students will enter **=a[] + b[]** to add Columns A and B, which calculates the number of tails for each trial. The possible numbers are 0, 1, and 2.

Note: Students can generate additional sets of data by choosing **MENU > Actions > Recalculate**. (ctrl + R).

To quickly determine the sums of 0, 1, and 2 tails, students are to use the **Quick Graph** feature. They need to make sure the label **total_tails** is on the lower part of the screen (x-axis). Students can move their cursors over each bar in the Bar Chart to see the number count for each outcome.

Students are to then calculate the experimental probabilities of tossing 0, 1, and 2 tails.

If the **Recalculate** command is used, the Quick Graph will change each time the command is executed.



TI-Nspire™ Navigator™ Opportunity: **Class Capture**
 See Note 1 at the end of this lesson.

Help students combine their results as a group, where the total number of trials is 100 multiplied by the number of students in the group, and as a class, where the total number of trials is 100 multiplied by the total number of students.

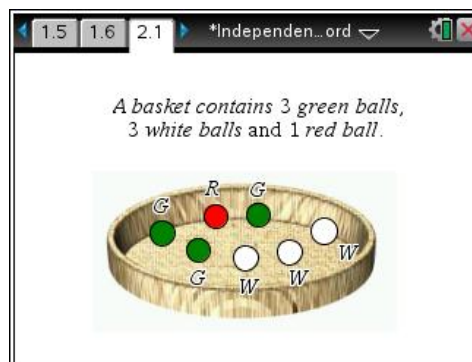
Students are to then compute the theoretical probabilities of tossing 0, 1, and 2 tails using the sample space. This is a good place to review the **Law of Large numbers**, where the experimental probability approaches the theoretical probability as the number of trials increases.

After answering the questions on page 2 of the worksheet, students should determine that if two events A and B are independent, then $P(A \text{ and } B) = P(A)P(B)$.

Problem 2 – One Independent and One Dependent

Students will now consider probabilities where the events are not both independent. They should understand that the first pick from the basket is an independent event, but the second event depends on the color of the first ball drawn because it was not replaced.

Explain to students that the probability of choosing a white ball after a red ball was already drawn from the basket is an example of conditional probability.



Problem 3 – Conditional Probability

Students are then given a spreadsheet on page 3.2 that contains the college enrollment (in thousands) for 2003. Students are given the formula for conditional probability on the worksheet. The questions will direct them towards a better understanding of how the formula is used to compute a probability. However, when the activity is finished a discussion on the formula for conditional probability is needed.

A screenshot of a TI-Nspire software window titled '*Independen...ord' showing a spreadsheet. The spreadsheet has columns for 'age', 'male', 'female', and 'total'. The data is as follows:

	age	male	female	total
1	15 - 17	61	89	150
2	18 - 21	3119	3926	7045
3	22 - 24	1578	1742	3320
4	25 - 29	982	1181	2163
5	30 - 34	607	723	1330
6	25+	970	1660	2630

The formula bar at the bottom shows $D1 = b1 + c1$.

Homework – Solutions

1. a. $\frac{1}{2}$
- b. Example: Let 0 = girl, 1 = boy. Use three lists to simulate 3 children. Do the simulation 100 times.
- c. Answers will vary.
- d. {GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB}
- e. $\frac{3}{8}$
- f. $\frac{2}{8} = \frac{1}{4}$

2. a. $\frac{1}{38}$

b. $\frac{18}{38} = \frac{9}{19}$

c. $\frac{9}{19} \cdot \frac{9}{19} = \frac{81}{361}$

d. $\frac{2}{38} = \frac{1}{19}$

e. $1 - \frac{1}{19} \cdot \frac{1}{19}$ or $\frac{18}{19} \cdot \frac{18}{19} = \frac{324}{361}$

3. One child – $\frac{1}{4}$

Both children – $\frac{1}{16}$

Neither child – $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

4. a. $\frac{13}{52} = \frac{1}{4}$

b. $\frac{4}{52} = \frac{1}{13}$

c. $\frac{1}{52}$

d. $\frac{12}{51} = \frac{4}{17}$

e. $\frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$

f. $\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} = \frac{33}{66,640} \approx 0.000495$

g. Four of a kind can be dealt 5 ways. The sum of the probabilities of the 5 ways is the total probability of a hand of 5 cards resulting in 4 of a kind.

$$P(1^{\text{st}} \text{ way}) = 1^{\text{st}} \text{ card drawn does not belong to the 4 of a kind} = \frac{52}{52} \cdot \frac{48}{51} \cdot \frac{3}{50} \cdot \frac{2}{49} \cdot \frac{1}{48}$$

$$P(2^{\text{nd}} \text{ way}) = 2^{\text{nd}} \text{ card drawn does not belong to the 4 of a kind} = \frac{52}{52} \cdot \frac{48}{51} \cdot \frac{3}{50} \cdot \frac{2}{49} \cdot \frac{1}{48}$$

$$P(3^{\text{rd}} \text{ way}) = 3^{\text{rd}} \text{ card drawn does not belong to the 4 of a kind} = \frac{52}{52} \cdot \frac{3}{51} \cdot \frac{48}{50} \cdot \frac{2}{49} \cdot \frac{1}{48}$$

$$P(4^{\text{th}} \text{ way}) = 4^{\text{th}} \text{ card drawn does not belong to the 4 of a kind} = \frac{52}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{48}{49} \cdot \frac{1}{48}$$

$$P(5^{\text{th}} \text{ way}) = 5^{\text{th}} \text{ card drawn does not belong to the 4 of a kind} = \frac{52}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} \cdot \frac{48}{48}$$

$$P(1) + P(2) + P(3) + P(4) + P(5) \approx 0.00024$$

TI-Nspire™ Navigator™ Opportunities

Note 1

Problem 1: Class Capture and/or Live Presenter

This would be a good place to do a Class Capture to compare student results. Class Capture may be used to compare students' graphs of the results of the simulation in Problem 1.