NUMB3RS Activity: A “Normal” Explosion
Episode: “Burn Rate”

Topic: Calculus: Derivatives **Grade Level:** 11 - 12
Objective: Students will determine the equations of normal lines.
Time: 15 - 20 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator

Introduction

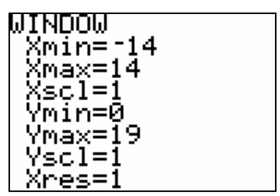
In “Burn Rate,” the FBI is investigating a series of mail bombs, and Charlie is called in to determine if any of the targets are related. When he visits the site of one of the detonations he begins to examine the debris and exclaims that the bomb was a high explosive, targeted shaped charge. “Classic Misznay-Schardin Effect—expanding gas and energy focused away from and perpendicular to the surface of the explosion.” By mathematically analyzing the dispersion of bomb fragments an estimate of the initial shape of the charge may be determined. One of the initial steps is to calculate the equations of the *normal lines* to possible bomb surfaces.

A normal line is a line that is perpendicular to a tangent line or plane. You can use these normal lines to determine the bomb fragment dispersion because when the velocity is high, as with an explosion, and the distance traveled is small, as in a room, the normal line approximates the parabolic path the debris follows. In this activity, students will calculate the equations of normal lines to a graph.

Discuss with Students

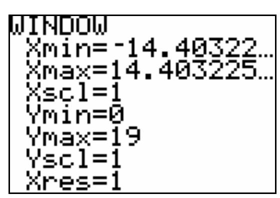
Finding the equation of a line tangent to a function and line normal to a function is a staple of a high school calculus course. This activity, however, provides a non traditional application of those concepts. Besides reviewing the derivative of functions, a quick review of finding the equation of a line may be useful to avoid any algebra mistakes.

When graphing the function and equations on a graphing calculator ensure that the students have a window large enough to display both the graph as well as the normal lines. It is also important that the window be set square so that the normal lines appear perpendicular to the function.



One suggestion is to use the window settings at the right.

After entering the minimum and maximum values, press ZOOM and select **5:ZSquare**. This will adjust the window setting to display a square window as shown at the right.



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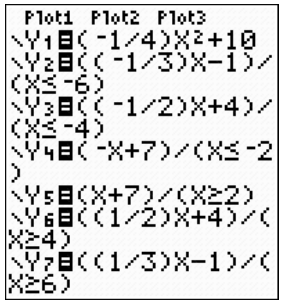
To remove clutter from the graph and have more relevance to the activity, students may graph the equations with domain restrictions. To do this, enter the equation enclosed in parenthesis divided by the domain also enclosed in parenthesis.

For the normal lines used in the activity:



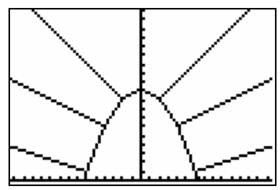
Student Page Answers:

1. *All of the graphs are either vertical or horizontal*. **2.** *All the graphs are either vertical or horizontal so the slopes are either zero or undefined.* **3.** *Most of the debris would hit the ceiling or the walls to either side of the charge. There would be very little in the upper corners of the room.* **4.** $y = -\frac{x}{2}$

5.

x	*y*	Tangent Slope	Normal Slope	Equation of Normal Line
–6	1	3	–1/3	*y* = –1/3*x* – 1
–4	6	2	–1/2	*y* = –1/2*x* + 4
–2	9	1	–1	*y* = –*x* + 7
0	10	0	Undefined	*x* = 0
2	9	–1	1	*y* = *x* +7
4	6	–2	1/2	*y* = 1/2*x* + 4
6	1	–3	1/3	*y* = 1/3*x* – 1

6.



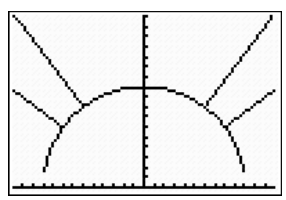
Extension Activity Answers:

1. *The derivative of the function is:* $y' = -x/y$

2.

x	*y*	Tangent Slope	Normal Slope	Equation of Normal Line
–10	0	Undefined	0	*y* = 0
–8	6	4/3	–3/4	*y* = –3/4*x*
–6	8	3/4	–4/3	*y* = –4/3*x*
0	10	0	Undefined	*x* = 0
6	8	–3/4	4/3	*y* = 4/3*x*
8	6	–4/3	3/4	*y* = 3/4*x*
10	0	Undefined	0	*y* = 0

3.



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Name: ________________________________ Date: _______________

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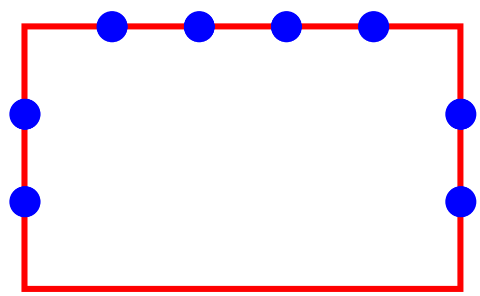
A normal line is a line that is perpendicular to a tangent line or plane. You can use these normal lines to determine the bomb fragment dispersion because when the velocity is high, as with an explosion, and the distance traveled is small, as in a room, the normal line approximates the parabolic path the debris follows. In this activity, you will calculate the equations of normal lines to a graph.

1. Examine a cross section of a block of explosive. Graph the normal lines from the surface.



2. How would you describe the slopes of these normal lines?

3. If the charge was placed in the center of a room, where you would expect most of the debris to be distributed about the room based upon the normal lines?

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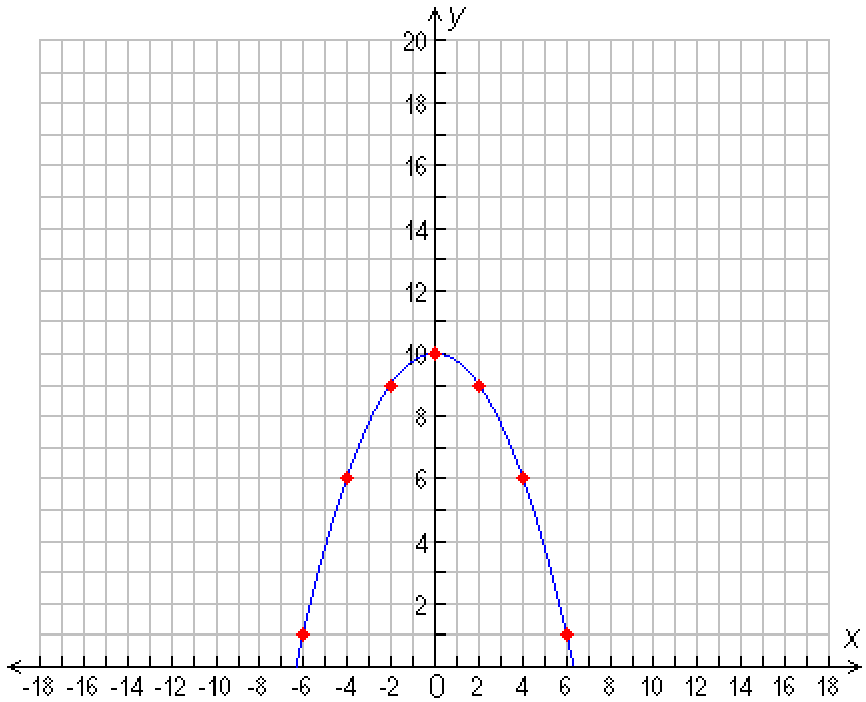
Examine an explosive charge formed in the shape of a parabola with the equation: $y = -\frac{1}{4}x^2 + 10$. Draw the normal lines on the graph below.



Find the equations of these normal lines. To do this you need two pieces of information: The slope of the line and the point. You have the points from the graph and you can determine the slope by evaluating the derivative of the function at the point.

4. Find the derivative of the function.

5. Use the derivative to complete the table below. Remember that the "Normal" slope is the opposite reciprocal of the slope of the tangent line.

x	*y*	Tangent Slope	Normal Slope	Equation of Normal Line
–6	1			
–4	6			
–2	9			
0	10			
2	9			
4	6			
6	1			

6. Use a graphing calculator to graph this information and compare to the sketch made earlier.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Another possible shape that the explosives may be formed into would be a hemisphere where: $x^2 + y^2 = 100$ where $y \geq 0$.

1. Using the implicit differentiation find the derivative of the function

2. Use the derivative to complete the table below.

x	***y***	**Tangent Slope**	**Normal Slope**	**Equation of Normal Line**
–10	0			
–8	6			
–6	8			
0	10			
6	8			
8	6			
10	0			

3. Use a graphing calculator to graph this information.

Additional Resources

For a Live Math example of finding the derivative of a quadratic function that you manipulate, go to: **http://archives.math.utk.edu/visual.calculus/2/definition.1/index.html**

This interactive applet from Dr. Barbara Kaskosz shows the graph of the derivative as the point is moved along the function.
http://www.math.uri.edu/~bkaskosz/flashmo/derplot

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