EXPLORATIONS™

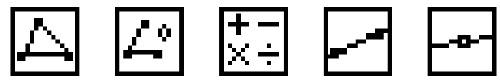
Activity 13

Angles of a Triangle

Objectives

- To investigate the sum of the angle measures of a triangle
- To investigate the relationship between an exterior angle and the interior angles of a triangle

Cabri® Jr. Tools



Introduction

Investigating the sum of the measures of the interior angles of a triangle is an essential activity for the understanding of geometry. In this activity, you will begin with an exploration of interior angles but will then investigate relationships between interior and exterior angles of a triangle.

This activity makes use of the following definitions:

Interior angles — angles formed at each vertex that lie inside the triangle.

Exterior angles — angles formed when the side of a triangle is extended. Exterior angles lie outside of the triangle and are adjacent to an interior angle.

Adjacent angles — angles with a common vertex and a common side, but no interior region in common.

Supplementary angles — two angles whose measures have a sum of 180°.

Remote interior angles — the interior angles of a triangle that are not adjacent to a given exterior angle.

Part I: Interior Angles of a Triangle

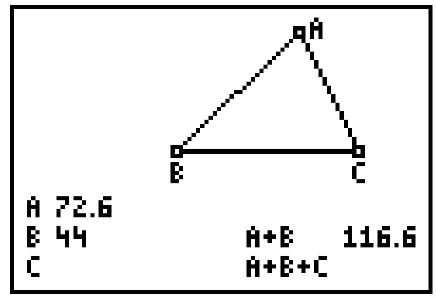
Construction

Construct a triangle and measure its interior angles.

Draw ΔABC near the center of the screen.

Measure and label each of the three interior angles.

Calculate the sum of the measures of the three angles.



Note: Not all measurements are shown.

Note: You can add only two measurements at a time using the Calculate *tool.*

Exploration

Observe the relationship among the angles as you change the triangle by dragging one of its vertices or one of its sides.

Questions and Conjectures

1. Make a conjecture about the sum of the interior angles of a triangle. Does your conjecture hold for any type of triangle (acute, right, obtuse, scalene, isosceles, and equilateral)? Explain your reasoning.
2. Describe a way to drag just a portion of ΔABC (a vertex or a side) so that exactly one angle remains constant. Describe a way that exactly two angles remain constant. Explain why this works and be prepared to demonstrate.

Part II: Exterior Angles of a Triangle

Construction

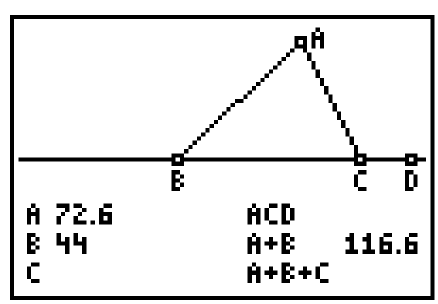
Construct an exterior angle to ΔABC.

Continue using the previous construction.

Use the Line tool to extend side $\overline{BC}$.

Construct a point D on $\overline{BC}$ as shown.

Measure $\angle ACD$.



Note: Not all measurements are shown.

Exploration

- Observe the relationship between $\angle ACB$ and $\angle ACD$ as you change ΔABC by dragging one of its vertices or one of its sides.
- Observe the relationship between $\angle ACD$, $\angle CAB$, and $\angle ABC$ as you change ΔABC by dragging one of its vertices or one of its sides.

Questions and Conjectures

1. Make a conjecture about the relationship between $\angle ACB$ and $\angle ACD$. Can this conjecture be extended to any exterior angle and its adjacent interior angle? Explain your reasoning and be prepared to demonstrate.
2. Make a conjecture about the relationship between an exterior angle and the remote interior angles. Would your conjecture be true for all types of triangles? Explain your reasoning.

Teacher Notes

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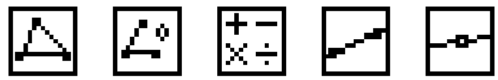
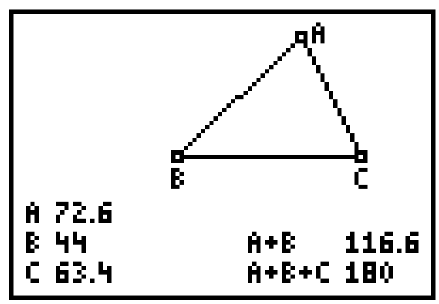


Part I: Interior Angles of a Triangle

Answers to Questions and Conjectures

1. Make a conjecture about the sum of the interior angles of a triangle. Does your conjecture hold for any type of triangle (acute, right, obtuse, scalene, isosceles, and equilateral)? Explain your reasoning.

   Dragging a vertex or side of the triangle should suggest that the sum of the angle measures of any triangle is equal to 180°.



2. Describe a way to drag just a portion of $\triangle ABC$ (a vertex or a side) so that exactly one angle remains constant. Describe a way so that exactly two angles remain constant. Explain why this works and be prepared to demonstrate.

   By dragging a vertex along an extension of that side, one angle will remain constant. For example, if side $\overline{BC}$ is horizontal and you drag vertex C horizontally, $m\angle B$ remains constant. If side $\overline{BC}$ is horizontal and you drag side $\overline{AC}$ horizontally, $m\angle C$ remains constant.

   There is no way to drag a side or a vertex that would have exactly two angle measures remaining constant. That would imply that the sum of the three angle measures was not constant.

Part II: Exterior Angles of a Triangle

Answers to Questions and Conjectures

1. Make a conjecture about the relationship between $\angle ACB$ and $\angle ACD$. Can this conjecture be extended to any exterior angle and its adjacent interior angle? Explain your reasoning and be prepared to demonstrate.

   Dragging a vertex of the triangle should suggest that the sum of the measure of the interior angle and its adjacent exterior angle is always equal to 180° since these two adjacent angles form a straight angle.

2. Make a conjecture about the relationship between an exterior angle and the remote interior angles. Would your conjecture be true for all types of triangles? Explain your reasoning.

   The measure of an exterior angle equals the sum of the measures of the non-adjacent (remote) interior angles. The reason for this relationship is a combination of the Angle Sum Theorem and the Supplementary Angles Theorem just explored. It can be shown that $m\angle ACD + m\angle ACB = m\angle CAB + m\angle ABC + m\angle ACB$ since both sums add to 180°. Subtracting $m\angle ACB$ from both sides gives the desired results.