Equations of Parabolas – ID: 10220

By Michele Patrick

Time required
60 minutes

Activity Overview

In this activity, students draw and measure lines and segments to discover properties of parabolas, specifically that the distance from any point on the parabola is equidistant to the focus and the directrix. When the point on the parabola is the vertex, the distance is related to the equation of the parabola.

Students work with parabolas whose vertex is on the origin as well as off the origin and they work with parabolas that open not only up and down, but also to the left and to the right. For these, they graph the parabola by writing two functions.

Students will derive general equation for parabolas, first where the vertex is at (0, 0), and then where the vertex is at (h, k).

Concepts

- *Focus and directrix of a parabola*
- *Line of symmetry*
- *Functions and nonfunctions*

Teacher Preparation

This activity is designed to be used in a Precalculus classroom. It can also be used in an Algebra 2 classroom.

- *Students should be familiar with the basic concepts of parabolas as graphs quadratic equations with a line of symmetry and a vertex. Students should also be able to write and solve quadratic equations.*
- *The screenshots on pages 2–5 demonstrate expected student results. Refer to the screenshots on pages 6 and 7 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student and solution .tns files and student worksheet, go to education.ti.com/exchange and enter "10220" in the quick search box.***

Classroom Management

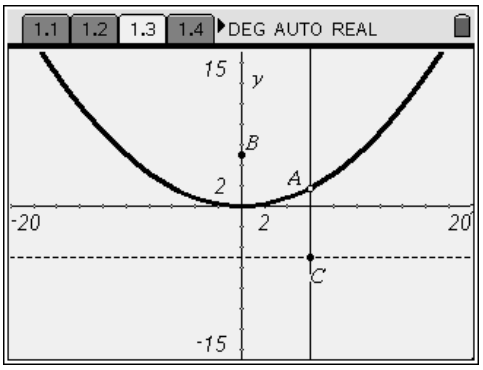
- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *The student worksheet PreCalcAct32_Parabolas_worksheet_EN provides a place for students to record their answers.*
- *The TI-Nspire solution document PreCalcAct32_Parabolas_Soln_EN.tns shows the expected results of working through the activity.*

TI-Nspire™ Applications

Graphs & Geometry, Notes

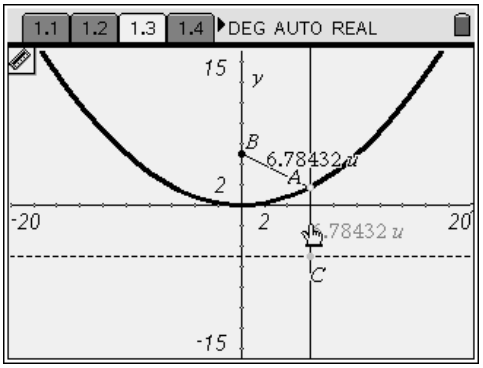
Problem 1 – Vertex at (0, 0)

Explain that every parabola has a focus and a directrix. The focus is a point on the line of symmetry and the directrix is perpendicular to the line of symmetry.



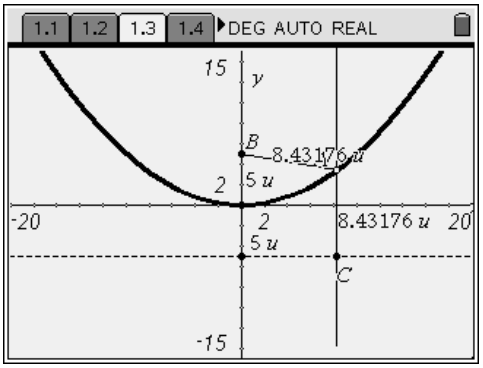
On page 1.3, have students draw a line through point *A* perpendicular to the directrix, (**MENU > Construction > Perpendicular**). Instruct students to find the point of intersection of the directrix and the perpendicular line (**MENU > Points & Lines > Intersection Point(s)**) and label it *C*.

Students will construct a segment (**MENU > Points & Lines > Segment**) from *A* to *B*, the focus, then find and compare *AB* and *AC*. To use the **Length** tool (**MENU > Measurement > Length**), select the endpoints of the segment to be measured.



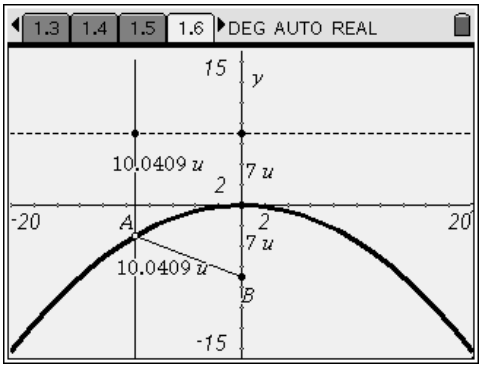
Students will grab and drag point *A* along the parabola. They should see that these lengths remain equal.

Next, students will find the distance from the vertex to the focus and from the vertex to the directrix. To do this, they will plot points at the intersections points between the *y*-axis and the parabola and the *y*-axis and the directrix.



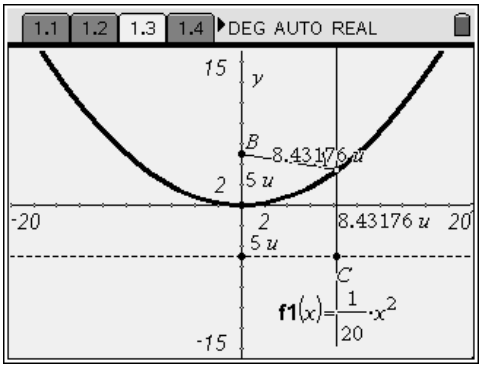
Students should see that these two distances are equal, which should not be a surprise, as the vertex is a point on the parabola and points on the parabolas are the same distance from the focus and directrix. The importance of this distance will be shown soon.

Let students work independently to find these same sets of measures for the parabola on page 1.6. They will again find two pairs of equal measures.

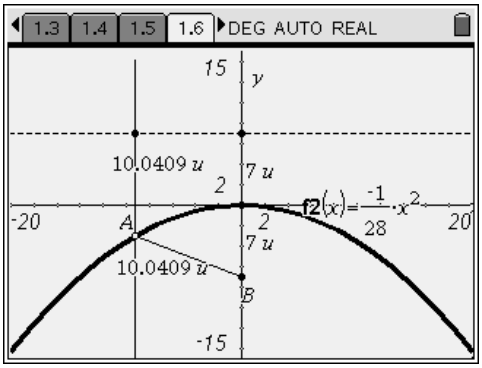


Students will use the **Hide/Show** tool (**MENU > Actions > Hide/Show**) to reveal the equation of the parabolas on pages 1.3 and 1.6.

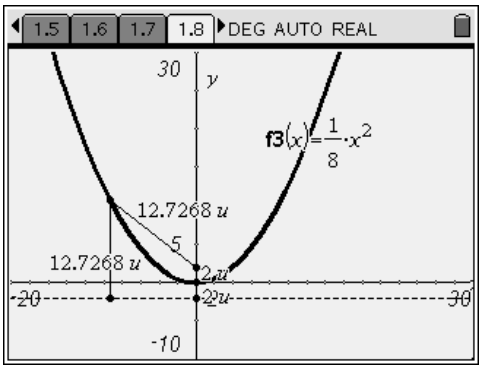
Give students a few minutes to consider the measures they found and how they might relate to the equations of the parabolas. They should conjecture how the equation can be found based on these measures.



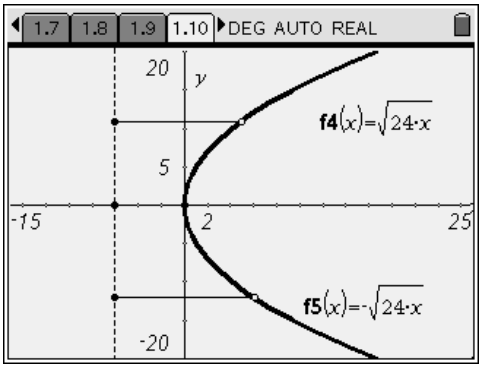
Let students share their ideas with the class. They should notice that the distance from the vertex to the focus (or directrix), multiplied by four, is equal to the denominator of the coefficient of the *x*-squared term.



Let students test their conjecture with the parabola and equation on page 1.8. Here, the distance from the vertex to directrix is two, and the denominator of the coefficient is four times two.

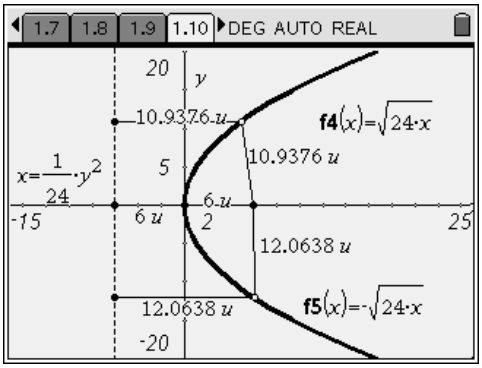


A parabola that opens to the right is shown on page 1.10. Ask students why there are two equations and have them write an equation for the parabola in terms of *y*. $\left(x = \frac{1}{24}y^2\right)$



Students will find the same measures they found for the other two parabolas to see if their conjectures apply to this parabola as well (yes).

Note that there are two points, one for each branch of the parabola.



Students should write general equations of a parabola using *p*, where the absolute value of *p* is the distance from the vertex to the focus. You can also ask for a general equation of the directrix.

Opening up: $y = \frac{1}{4p}x^2$, $p > 0$; Opening down: $y = \frac{1}{4p}x^2$, $p < 0$;

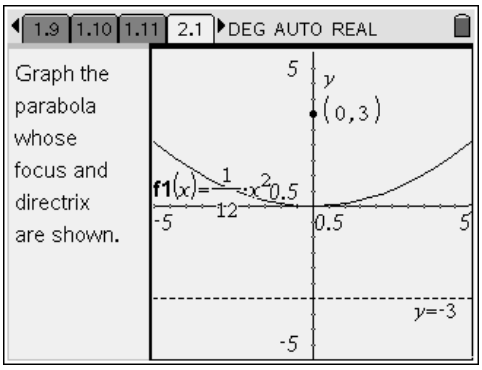
Focus: (0, *p*); Directrix: $y = -p$

Opening right: $x = \frac{1}{4p}y^2$, $p > 0$; Opening left: $x = \frac{1}{4p}y^2$, $p < 0$;

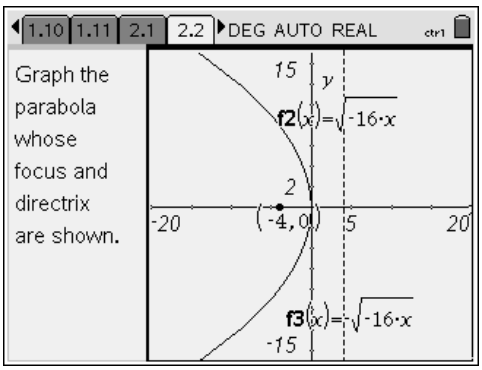
Focus: (*p*, 0); Directrix: $x = -p$

Problem 2 – Practice

Let students work independently through pages 2.1 and 2.2. For each, the focus and directrix are shown. Walk around and assist students as needed.



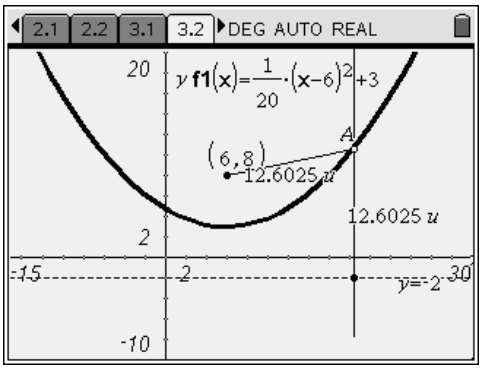
On page 2.2, students will need to enter two equations to graph the parabola.



Problem 3 – Vertex at (*h*, *k*)

The parabolas in Problem 3 do not have a vertex located at (0, 0). The coordinates of the vertex are (*h*, *k*).

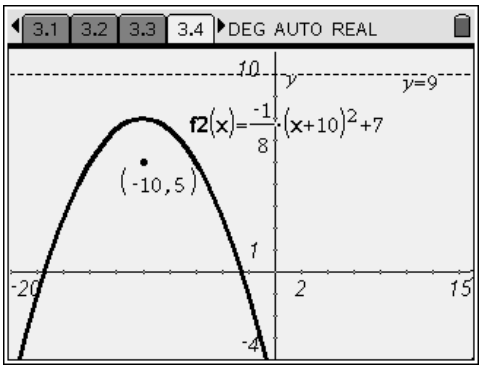
Tell students to move to page 3.2 and confirm that the distance from point *A* to the focus is equal to the distance from point *A* to the directrix.



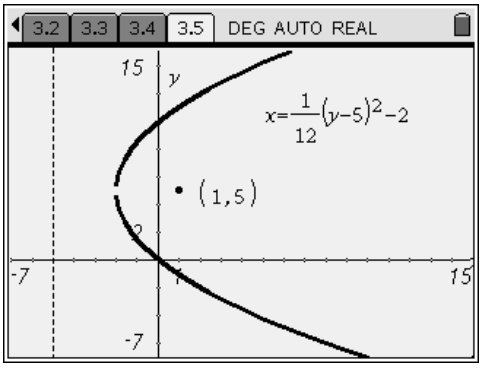
Ask students how they can find *p*, the distance from the vertex to the focus (either divide the denominator in the equation of the parabola by four or find the distance manually, remembering that both the focus and vertex are on the line of symmetry). For the parabola on page 3.2, $p = 5$.

Have students conjecture how *p* can be related to the coordinates in the focus and directrix.

Let students work independently, or in pairs, to analyze the values of *p*, *h*, and *k* in the next two diagrams. They should write general equations for a parabola whose vertex is at (*h*, *k*) and general expressions for the focus and directrix for these parabolas.



Students can find the coordinates of the focus by selecting **MENU > Actions > Coordinates and Equations**.



Opening up or down: $y = \frac{1}{4p}(x-h)^2 + k$

Focus: (h, $k + p$), Directrix: $y = k - p$

Opening right or left: $x = \frac{1}{4p}(y-k)^2 + h$

Focus: ($h + p$, k), Directrix: $x = h - p$

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(Student)TI-Nspire File: *PreCalcAct32_Parabolas_EN.tns*