

Exploring Polar Equations Two of the basic forms of a polar curve are given by  $r(\theta) = a + b \cos(n\theta)$  and  $r(\theta) = a + b \sin(n\theta)$ . By changing the values of *a*, *b*, and *n* different polar curves can be generated.

## **Circle Exploration**

- 1. Open a new TI InterActive! document. Title this document **Exploring Polar Equations**. Add your name and the date to this document.
- 2. Select Mode settings and change the Graph Type to Polar and the Angle Format to Radian. Click on OK.
- 3. Select Graph P to open a Graph window and define  $r1(\theta) := 2 \cos(\theta)$ . Click in the box to the left of  $r1(\theta)$  to select the equation. Sketch the graph of  $r1(\theta) := 2 \cos(\theta)$  on the provided grid.

6 4 2 -10 -8 10 -6 -4 -2 4 6 8 -2 -4 -6 -8 -10

10

8

Note: Use the Symbol Palette to access the  $\theta$ .

- 4. Define and select  $r2(\theta)$ : =  $3\cos(\theta)$ . Sketch the graph of  $r2(\theta)$ : =  $3\cos(\theta)$  on the same grid.
- 5. Define and select  $r_3(\theta)$ : = -4 cos( $\theta$ ). Sketch the graph of  $r_3(\theta)$ : = -4 cos( $\theta$ ) on the same grid.

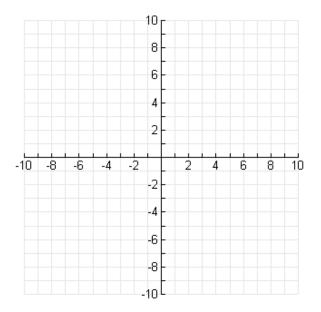
- 6. Define and select  $r4(\theta)$ : =  $-5 \cos(\theta)$ . Sketch the graph of  $r4(\theta)$ : =  $-5 \cos(\theta)$ on the same grid.
- Ŧ 7. Click on Save to Document to paste the graphs into your TI InterActive! document.

# **Circle Analysis**

- 1. In the equation  $y = a + b \cos(n\theta)$ , what is the value of *a* for each of the equations in *Circle Exploration* questions 3 through 6? What is the value of *n*?
  - *n* = \_\_\_\_\_ a =
- 2. What effect does the absolute value of b have on the graph of the circle?
- 3. What effect does the positive or negative value of *b* have on the graph?

# **Rose Polar Curve Exploration**

- 1. Select Graph 😢 to open a new graphing window and define  $r1(\theta)$ : = 4 cos( $\theta$ ). Click in the box to the left of  $r1(\theta)$  to select the equation. Sketch the graph of  $r1(\theta)$ : = 4 cos( $\theta$ ) on the provided grid.
- 2. Define and select  $r2(\theta)$ : = 4 cos(2 $\theta$ ). Sketch the graph of  $r2(\theta)$ : = 4 cos(2 $\theta$ ) on the same grid.
- 3. Define and select  $r_3(\theta)$ : = 4 cos(3 $\theta$ ). Sketch the graph of  $r3(\theta)$ : = 4 cos(3 $\theta$ ) on the same grid.



- 4. Define and select  $r4(\theta)$ : = 4 cos(4 $\theta$ ). Sketch the graph of  $r4(\theta)$ : = 4 cos(4 $\theta$ ) on the same grid.
- 5. Click on Save to Document to paste the graphs into your TI InterActive! document.

### **Rose Polar Curve Analysis**

1. In the equation  $y = a + b \cos(n\theta)$ , what is the value of *a* for each of the equations in *Rose Polar Curves Exploration* questions 1 through 4? What is the value of *b*?

*a* = \_\_\_\_\_ *b* = \_\_\_\_\_

2. How many rose leaves does each equation produce?

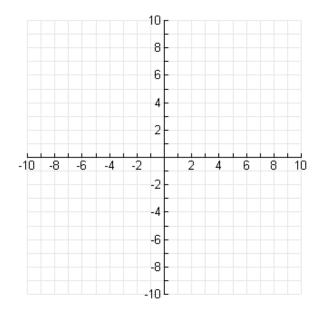
A.  $r1(\theta) := 4 \cos(\theta)$  leaves = \_\_\_\_\_

- B.  $r_{2}(\theta) := 4 \cos(2\theta)$  leaves = \_\_\_\_\_
- C.  $r3(\theta) := 4 \cos(3\theta)$  leaves = \_\_\_\_\_
- D.  $r4(\theta) := 4 \cos(4\theta)$  leaves = \_\_\_\_\_
- 3. How does the value of *n* determine the number of leaves?
- 4. Select Graph P to open a new graph window. Define  $r1(\theta): = 4 \cos(3\theta), r2(\theta): = 5 \cos(3\theta)$  and  $r3(\theta): = 6 \cos(3\theta)$ . Click on Save to Document P to paste the graphs into your TI InterActive! document. What effect does the value of *b* have on the leaves of the rose?
- 5. Select Graph 🔄 to open a new graph window. Define

 $r1(\theta)$ : = 5 cos(3 $\theta$ ) and  $r2(\theta)$ : = -5 cos(3 $\theta$ ). Click on Save to Document to paste the graphs into your TI InterActive! document What effect does the positive or negative value of *b* have on the graph?

# Limaçon Curve Exploration

- 1. Select Graph P to open a new graphing window and define  $r1(\theta)$ : = 1 + 2 cos( $\theta$ ). Click in the box to the left of  $r1(\theta)$  to select the equation. Sketch the graph of  $r1(\theta)$ : = 1 + 2 cos( $\theta$ ) on the provided grid.
- 2. Define and select  $r2(\theta)$ : = 2 + 4 cos( $\theta$ ). Sketch the graph of  $r2(\theta)$ : = 2 + 4 cos( $\theta$ ) on the same grid.
- 3. Define and select  $r_3(\theta)$ : = 1 - 3 cos( $\theta$ ). Sketch the graph of  $r_3(\theta)$ : = 1 - 3 cos( $\theta$ ) on the same grid.



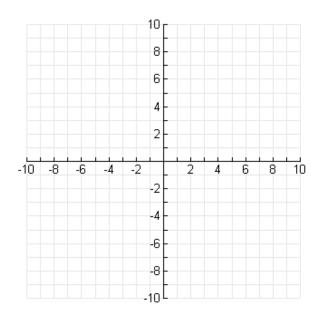
- 4. Define and select  $r4(\theta)$ : = 2 5 cos( $\theta$ ). Sketch the graph of  $r4(\theta)$ : = 2 5 cos( $\theta$ ) on the same grid.
- 5. Click on Save to Document to paste the graphs into your TI InterActive! document.

## Limaçon Curve Analysis

- 1. In the equation  $y = a + b \cos(n\theta)$ , what is the value of *n* for each of the equations in *Limaçon Curve Exploration* questions 1 through 4?
  - *n* = \_\_\_\_\_
- 2. How does the absolute value of *a* compare to the absolute value of *b*?
- 3. How do the absolute values of *a* and *b* effect the graph?
- 4. What effect does the positive or negative value of *b* have on the graph?

### **Cardiod Curve Exploration**

- 1. Select Graph to open a new graphing window and define  $r1(\theta)$ : = 2 + 2 cos( $\theta$ ). Click in the box to the left of  $r1(\theta)$  to select the equation. Sketch the graph of  $r1(\theta)$ : = 2 + 2 cos( $\theta$ ) on the provided grid.
- 2. Define and select  $r2(\theta)$ : = 3 + 3 cos( $\theta$ ). Sketch the graph of  $r2(\theta)$ : = 3 + 3 cos( $\theta$ ) on the same grid.
- 3. Define and select  $r3(\theta)$ : = 4 + 4 cos( $\theta$ ). Sketch the graph of  $r3(\theta)$ : = 4 + 4 cos( $\theta$ ) on the same grid.



- 4. Define and select  $r4(\theta)$ : = 5 + 5 cos( $\theta$ ). Sketch the graph of  $r4(\theta)$ : = 5 + 5 cos( $\theta$ ) on the same grid.
- 5. Click on Save to Document to paste the graphs into your TI InterActive! document.

## **Cardiod Curve Analysis**

1. In the equation  $y = a + b \cos(n\theta)$ , what is the value of *n* for each of the equations in *Cardiod Curve Exploration* questions 1 through 4?

*n* = \_\_\_\_\_

- 2. How does the absolute value of *a* compare to the absolute value of *b*?
- 3. How do the absolute values of *a* and *b* affect the graph?
- 4. Select Graph 1 to open a new graph window. Define  $r1(\theta) := 3 + 3 \cos(\theta)$ and  $r2(\theta) := 3 - 3 \cos(\theta)$ . Click on Save to Document 1 to paste the graphs into your TI InterActive! document. What effect does the positive or negative value of *b* have on the graph?
- 5. Save this document as **cardiod.tii**. Print a copy of this document.

### Additional Exercises

In a math box, define a: = 0, b = 2, n: = 1. Select Polar Graph and define  $r1(\theta)$ : = a + b \* sin(n \*  $\theta$ ). Describe the graph produced and indicate whether the graph is a circle, rose, limaçon or cardiod. Change the values to those given in each problem.

- 1. a: = 0, b: = 2, n: = 1
- 2. a: = 0, b: = 4, n: = 1
- 3. a: = 0, b: = -6, n: = 1
- 4. a: = 0, b: = 2, n: = 2
- 5. a = 0, b = 2, n = 3
- 6. a = 0, b = -3, n = 4
- 7. a = 1, b = 2, n = 1

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8. a: = 2, b: =2, n: = 1

- 9. a: = 3, b: = -4, n: = 1
- 10. a: = 2, b: = -5, n: = 1
- 11. How do the curves  $r(\theta) = a + b \cos(n\theta)$  and  $r(\theta) = a + b \sin(n\theta)$  compare?
- 12. Generalize how the values of *a*, *b*, and *n* produce the different curves.