MATRICES AND LINEAR EQUATIONS

A matrix is a rectangular pattern of elements arranged in rows and columns. We normally label a matrix with a capital letter A, B, C,..... and we usually describe a matrix by it number of rows, *m*, and its number of columns, *n*, hence a *m* x *n* matrix.

eg. $A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is a 2 x 2 matrix and $B = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ is a 2 x 1 matrix.

A true matrix has all columns and rows complete.

ADDITION AND SUBTRACTION OF MATRICES

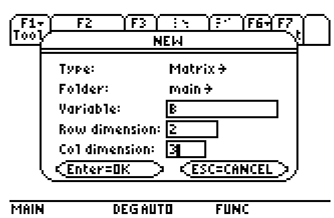
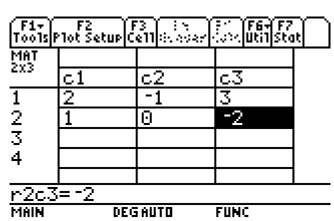
Only matrices of the same size can be added or subtracted.

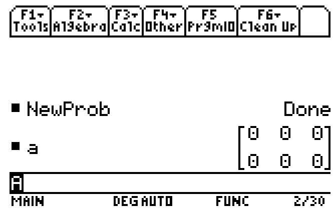
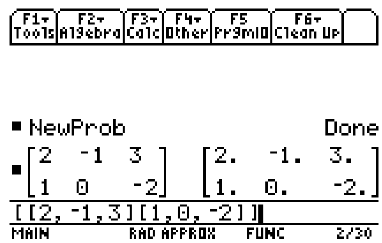
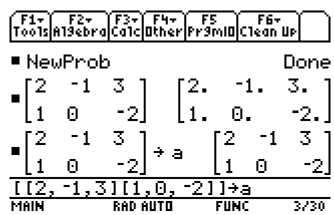
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, then $A + B = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

Note that each corresponding element is added together. This of course would also work for subtraction.

ON YOUR CALCULATOR

<table>
<tr><td>In the APPS menu select
6: Data/Matrix Editor

Then select
3: New</td><td></td></tr>
<tr><td>Under Type select 1: Data
Use a letter to name the matrix
Set the row and column dimensions</td><td>
</td></tr>
<tr><td>Use the editor window to enter the matrix.</td><td>
</td></tr>
</table>

If we now go to the HOME screen we can type in the name of the matrix and it will be printed in matrix form.	
From the HOME screen you can type in a matrix directly: Note here you are entering the matrix row by row.	
This can then stored as a letter: Just use the store button and then an appropriate letter.	

PRODUCT OF A MATRIX AND A SCALAR

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $2A = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$

eg. If $A = \begin{bmatrix} 0 & 1 \\ 3 & -1 \end{bmatrix}$ then $3A = \begin{bmatrix} 0 & 3 \\ 9 & -3 \end{bmatrix}$

THE UNIT MATRIX

The square unit matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called the identity 2 x 2 matrix and can be denoted by I.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the unit or identity 3 x 3 matrix.

Exercise 1:

1. If $A = \begin{bmatrix} 4 & -6 \\ -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 8 \\ -4 & 6 \end{bmatrix}$ find:

(i) A + B (ii) A - B (iii) -2A

MULTIPLICATION OF MATRICES:

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, then $AB = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$

eg. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ then

$$AB = \begin{bmatrix} 1x5 + 2x7 & 1x6 + 2x8 \\ 3x5 + 4x7 & 3x6 + 4x8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

Matrices do not need to be the same size to be able to multiply.

If the first matrix is an *m x p* and the second matrix is *p x n* then the product will be a *m x n* matrix. Note the number of columns in the first matrix must equal to the number of rows in the second matrix.

Exercise 2:

1. Find the matrix products in the following questions:

(i) $\begin{bmatrix} 2 & 5 \\ -3 & -4 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 4 & 3 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \end{bmatrix}$

(iii) $\begin{bmatrix} -6 & -4 & 2 \\ 7 & 8 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} -6 & -4 & 2 \\ 7 & 8 & -5 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -2 & 2 \\ -3 & 1 \end{bmatrix}$

(vi) $\begin{bmatrix} -2 & 2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

INVERSE MATRICES

An important aspect of matrices is the ability to be able to find the inverse of the matrix. This replaces the idea of division.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the first step to finding its inverse is to calculate the determinant. The determinant is represented by $\Delta = ad - bc$. The inverse of A is then given by:

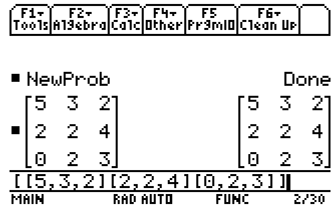
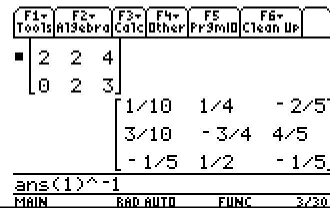
$$A^{-1} = \frac{1}{\Delta}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \text{or} \quad A^{-1} = \begin{bmatrix} \frac{d}{\Delta} & \frac{-b}{\Delta} \\ \frac{-c}{\Delta} & \frac{a}{\Delta} \end{bmatrix}$$

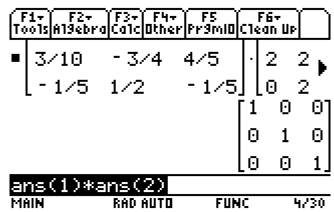
eg. If $A = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ then $\Delta = 5x4 - 7x2 = 6$

Then $A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & -2 \\ -7 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{6} & \frac{-2}{6} \\ \frac{-7}{6} & \frac{5}{6} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-7}{6} & \frac{5}{6} \end{bmatrix}$

Then we can check what the advantage of the inverse is:

$A \times A^{-1} = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-7}{6} & \frac{5}{6} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ The Unit matrix.

<table>
<tr><td>Type in the matrix you are working with:</td><td>
</td></tr>
<tr><td>Use the ANS feature to find the inverse:</td><td>
</td></tr>
<tr><td>Check that when you multiply the original and the inverse together that the unit matrix is produced.

Note ans(2) is the second last answer!</td><td>
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Exercise 3:

1. Find the inverse of the following matrices:

(i) $\begin{bmatrix} -4 & 5 \\ 3 & -4 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} p & q \\ 0 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 5 & 3 & 2 \\ 2 & 2 & 4 \\ 0 & 2 & 3 \end{bmatrix}$

This leads us into the realm of simultaneous equations:

Consider the problem of solving the set of simultaneous equations:

$$\begin{aligned}5x+4y&=2\\3x+2y&=0\end{aligned}$$

This can be represented as $\begin{bmatrix}5 & 4\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}$

The purpose of solving these equations is to find the values of x and y. Therefore we need to remove the matrix at the front of the column matrix. This can be done by multiplying by the inverse of $\begin{bmatrix}5 & 4\\3 & 2\end{bmatrix}$. Using the method above or by using your calculator find the inverse which is $\begin{bmatrix}-1 & 2\\\frac{3}{2} & \frac{-5}{2}\end{bmatrix}$.

$$\begin{bmatrix}-1 & 2\\\frac{3}{2} & \frac{-5}{2}\end{bmatrix}\begin{bmatrix}5 & 4\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-1 & 2\\\frac{3}{2} & \frac{-5}{2}\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2\\3\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2\\3\end{bmatrix}$$

Therefore the solution is x = -2 and y = 3.

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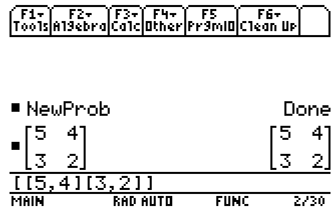
<tr><td>Type in the matrix you are working with:</td><td>
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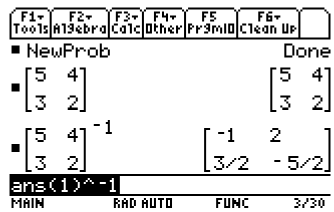
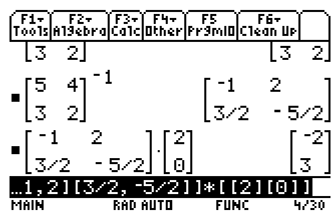
<tr><td>Use the ANS feature to find the inverse:</td><td>
</td></tr>
<tr><td>Multiply the inverse by the answer matrix.</td><td>
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This of course means we can solve very complex sets of simultaneous equations with ease.

Exercise 4:

1. Solve the following sets of simultaneous equations:

(i)
$$\begin{aligned} 3x + 4y &= 4 \\ x - 2y &= 18 \end{aligned}$$

(ii)
$$\begin{aligned} x + 2y + 3z &= 1 \\ 2x + 4y + 5z &= 6 \\ 3x + 5y + 6z &= -6 \end{aligned}$$

(iii) At a snack bar John paid \$5.15 for a hamburger, a dim sim and a serve of chips. At the same snack bar Andrew paid \$6.50 for a hamburger, 4 dim sims and a serve of chips while it cost Mr Thomson \$21.20 for 4 hamburgers, 8 dim sims and 3 serves of chips. Find the cost of each food item.