NUMB3RS Activity: Bat Sonar
Episode: "Guns and Roses"

Topic: Echolocation in bats **Grade Level:** 7 - 12

Objective: To use algebra to model bat sonar (echolocation)

Time: 10 - 15 minutes

Introduction

When ATF officer Nikki Davis apparently commits suicide, the sound of the gunshot was recorded from multiple sources. Charlie's attempts to recreate the "acoustic fingerprint" of the shot are unsuccessful until he realizes that the key may lie in the way bats use sounds to "see." A bat emits a sound wave, and by hearing how long it takes the echo to return, its brain automatically knows how far away an object is.

This activity allows students to study how this process works using an algebraic model.

Discuss with Students

Bat sonar, called "echolocation," is a widely studied phenomenon. It is not unique to bats. Many species, including some fish, use a similar process. Bats emit an ultrasonic "squeak," each about 1 – 2 milliseconds (ms) in duration, at a rate of 20 – 60 squeaks per second, at frequencies of 20 kHz – 100 kHz (kHz is "kilohertz," or thousands of cycles per second). By comparison, humans can hear frequencies that range from 20 Hz to 20 kHz, and the human voice has a frequency of about 1 kHz.

Each bat species has its own "vocal signature" (which is similar to how Charlie tries to recreate the acoustic fingerprint of the crime scene). For example, researchers in Belize are able to identify 80% of the known bat species there just by their voices.

This activity focuses on the use of the formula $d = vt$ (sometimes written as $d = rt$), where v is the speed of sound, d is twice the distance between the bat and another object, and t is the time it takes for the echo of the bat's squeak to travel from the bat to the object and return.

The speed of sound used is this activity (1,115 ft/sec) is actually for an air temperature of about 58°F (which is a good bug-hunting temperature). In general, the speed of sound is determined by the formula $V = 1052 + 1.08T$, where T is the temperature in degrees Fahrenheit.

Student Page Answers:

1. *Total distance traveled by the sound is 100 ft, so* $t = \frac{100}{1{,}115} \approx 0.0897$ *sec. Note: This is a good spot to discuss appropriate numbers of significant digits in answers.* **2.** *d = (0.04)(1,115) = 44.6 ft Because the sound traveled both ways (to and from the bug), the bug is 22.3 ft away.* **3.** *Using 5280 ft/mi and 3600 sec/hr yields 36.666… ≈ 36.7 ft/sec* **4a.** *(0.08968)(1,115) = 100.0 ft (total, so the actual distance is 50 ft)* **4b.** *(0.08682)(1,115) = 96.8 ft (again, total, so the distance is 48.4 ft)* **4c.** *50.0 – 48.4 = 1.6 ft.* **4d.** *Using a proportion,* $\frac{1.6}{0.04} = \frac{v}{1}$*, so the bat is flying at 40 ft/sec, or about 27.3 mi/hr.*

L. Charles (Chuck) Biehl, The Charter School of Wilmington, Wilmington, DE

Name: ________________________________ Date: ______________

NUMB3RS Activity: Bat Sonar

When ATF officer Nikki Davis apparently commits suicide, the sound of the gunshot was recorded from multiple sources. Charlie's attempts to recreate the "acoustic fingerprint" of the shot are unsuccessful until he realizes that that the key may lie in the way bats use sounds to "see." A bat emits a sound wave (an ultrasonic "squeak") and by hearing how long it takes the echo to return, its brain automatically knows how far away an object (like an animal or a bug) is. This process is called "echolocation."

The mathematical model used for echolocation should be a familiar one: $d = vt$, which relates distance, velocity (speed with direction), and time. Because the bat operates on differences in time, we can also use $t = \frac{d}{v}$. In this case, *v* is the speed of sound, *d* is twice the distance between the bat and another object, and *t* is the time it takes for the echo of the bat's squeak to travel from the bat to the object and return.

1. If a bat is stationary, how long does it take to hear the echo from an animal 50 feet away? (Use 1,115 ft/sec for the speed of sound.)

2. How far away is a bug if its echo returns in 0.04 seconds?

Here are some "bat facts":

- Bat squeaks are ultrasonic, usually in the range of 20 kHz – 100 kHz (a human voice is about 1 kHz)
- Bats squeak 20 – 60 times per second, each one lasting about 1 millisecond
- Each species has its own unique "vocal signature." Many species can be identified by vocal signature alone (similar to the "acoustic fingerprint" discussed in "Guns and Roses")
- Some bats can fly as fast as 60 mi/hr; the average speed while hunting for food is closer to 25 mi/hr.

3. Because the speed of sound has been listed in ft/sec, use dimensional analysis (sometimes called the *factor label method* or *unit conversion method*) to convert average flying speed to ft/sec. Fill in the blanks, canceling units as you go:

$$25\frac{\text{mi}}{\text{hr}} \times \frac{\underline{\qquad}\text{ft}}{\text{mi}} \times \frac{\text{hr}}{\underline{\qquad}\text{sec}} = \underline{\qquad} \frac{\text{ft}}{\text{sec}}.$$

4. Suppose a certain bat squeaks 25 times per second (once every 0.04 sec.), and is flying toward a bug on the ground. On two consecutive squeaks, the echo return times are 0.08968 sec. and 0.08682 sec. respectively. (Express answers to the nearest tenth, and remember that the speed of sound is 1,115 ft/sec)

a. How far away is the bug after the first squeak?
b. How far away is the bug after the second squeak?
c. How far does the bat travel between squeaks?
d. How fast is the bat flying in ft/sec and mi/hr?

L. Charles (Chuck) Biehl, The Charter School of Wilmington, Wilmington, DE

The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Even from this simple activity it is clear that a bat's echolocation mechanism is extremely complex and sophisticated. Scientists believe that bats use their sonar to generate a three dimensional "image" of objects, by not only knowing the direction and distance of objects in their sonar, but also reconstruct their shapes, and do both simultaneously.

For the Student

In this activity, we considered the bat flying toward a stationary bug. Using the activity as a guide, investigate how you could mathematically model the following bat/bug scenarios, using the "consecutive squeak" point of view:

- The bug is approaching the bat.
- The bug and the bat are moving toward each other.
- The bug and the bat are moving away from each other.
- The bug and the bat are moving in the same direction at different speeds.

What information would you have to know in order to model each situation? How little information would you need in order to be able to calculate velocity, distance, and/or location for the bat and the bug?

Related Topic

Another aspect of echolocation is that of Doppler shift. You may know that when a train or emergency vehicle passes by, you can hear changes in the pitch of the sound based on how close you are to the source of the sound. Even though bats use ultrasonic squeaks, if the bat and bug are moving toward each other, the echo will come back at a higher frequency than transmitted (convince yourself that this is true). Similarly, if the bug is trying to escape, the echo will return at a lower frequency.

Additional Resources

For an informative article regarding the development of sonar in bats and other animals, as well as some of the DNA and fossil evidence to support its evolution, visit the Science News Online Web Site.
http://www.sciencenews.org/articles/20050514/bob9.asp

For a detailed discussion of bat echolocation, including Doppler shift, visit this Web site.

http://nelson.beckman.uiuc.edu/courses/neuroethol/models/bat_echolocation/bat_echolocation.html

This article is from the field of neuroethology, which is the study of how nervous systems generate natural behavior in animals. Because bats have nervous systems highly dedicated to echolocation, they are an excellent species to study. It is also the reference that was used by the writers for this episode of *NUMB3RS*.

Visit the Web site below to download a CBC radio program about bat echolocation.
http://www.cbc.ca/quirks/archives/05-06/mar25.html

L. Charles (Chuck) Biehl, The Charter School of Wilmington, Wilmington, DE