Capacitors – ID: 9460

By Charles W. Eaker

Time required
45 minutes

Activity Overview

In this activity, students explore the characteristics of capacitors. The activity begins with a look at the movement of charge in a parallel-plate capacitor and the effect of voltage on charge. Following the definition of capacitance, students explore how plate area, plate separation, and dielectrics affect capacitance. The activity ends with an investigation of the rate of discharge of a capacitor in an RC circuit.

Concepts

- *Storage of charge in a capacitor*
- *Physical characteristics of capacitors*
- *Rate of discharge of capacitors*

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes, Data & Statistics, Lists & Spreadsheet

Teacher Preparation

Before carrying out this activity, review electrical units such as ohms, volts, coulombs, and farads with students. Review the concept of a capacitor with them.

- *The screenshots on pages 2–7 demonstrate expected student results. Refer to the screenshots on pages 8–10 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9460" in the search box.***

Classroom Management

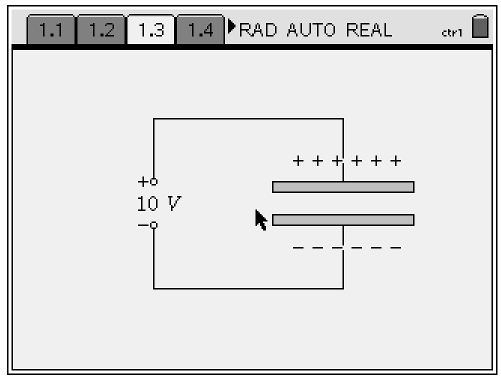
- *This activity is designed to be* ***teacher-led*** *with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in* ***this*** *document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration in this activity:

- What are a capacitor and capacitance?
- What determines the capacitance of a capacitor?
- What is the role of a dielectric in a capacitor?
- How fast do capacitors discharge?

Students will vary the area, *A*, of the plates and also the separation, *d*, of the plates in a parallel-plate capacitor and will develop an equation for capacitance, *C*, in terms of *A* and *d*. Students will construct plots of charge vs. voltage and voltage vs. time for capacitors. Using experimental data, students will determine the capacitance of an unknown capacitor in an *RC* circuit.

Part 1 – Capacitors



Step 1: Students should open the file **PhyAct16_capacitors_EN.tns** and read the first two pages. Page 1.3 illustrates a charged capacitor. Students should examine the capacitor on page 1.3 and then answer questions 1–3.

Q1. If the charge on the negative plate is –Q, what is the charge on the positive plate?

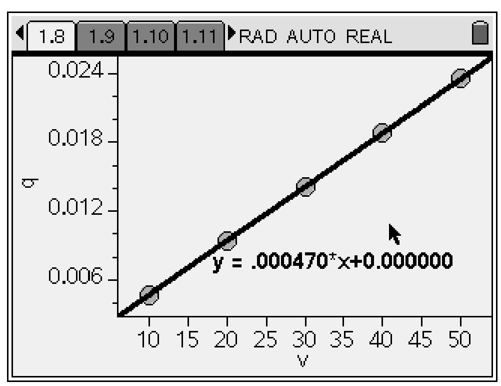
A. *+Q*

Q2. What is the total net charge of the capacitor?

A. *zero*

Q3. If the potential difference between the plates is increased from 10 V to 20 V, will Q increase, decrease, or be unchanged?

A. *The charge will increase.*



Step 2: Next, students should read page 1.6 before examining the *Lists & Spreadsheet* application on page 1.7, which contains data on charge (**q**) and voltage (**v**) for a capacitor. On page 1.8, they should make a scatter plot of the data, using **v** as the *x*-variable and **q** as the *y*-variable. Students should then use the **Linear Regression** tool (**Menu > Actions > Regression > Show Linear (mx + b)**) to find an equation relating **v** and **q**. They should then answer questions 4–6. Note: Make sure students read the text on page 1.10 before answering question 6.

Q4. Describe the relationship between charge (**q**) and potential (**v**).

A. *Charge increases linearly with electrical potential.*

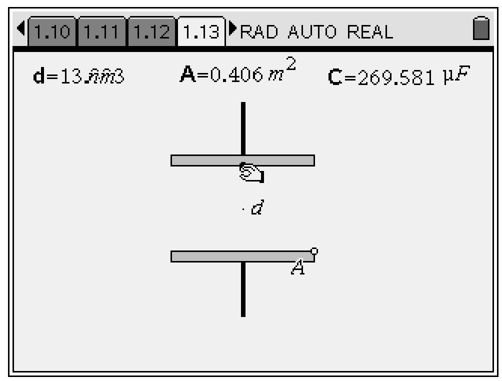
Q5. Write an equation relating **q** and **v**.

A. $\boldsymbol{q} = (0.000470)\boldsymbol{v}$

Q6. What is the capacitance in μF of the capacitor graphed on page 1.8?

A. *470 μF; you may need to review the conversion of farads to microfarads with students.*

Part 2 – Physical characteristics of capacitors



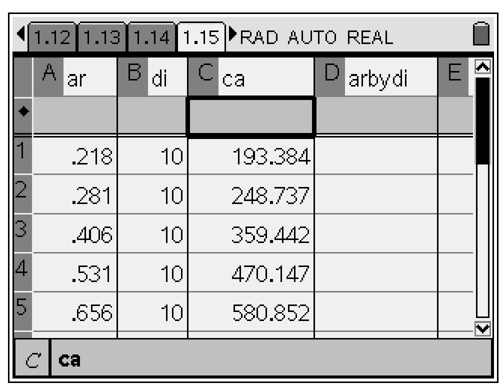
Step 1: Next, students should read the text on page 1.12 before moving to page 1.13, which shows a parallel-plate capacitor. Students can vary the separation and the area of the plates and observe the effects on capacitance. They can do this by dragging the black point (plate separation) up and down and the white point (plate area) left and right. After students have manipulated the plate area and separation, they should answer questions 7 and 8.

Q7. How does increasing the plate area affect capacitance?

A. *Increasing plate area increases capacitance.*

Q8. How does decreasing plate separation affect capacitance?

A. *Decreasing separation increases capacitance.*



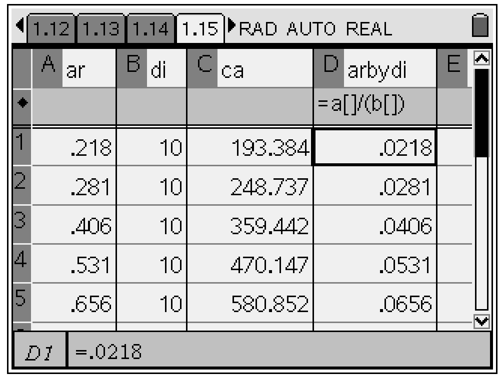
1.12 1.13 1.14 1.15 RAD AUTO REAL

	A ar	B di	C ca	D arbydi	E
◆					
1	.218	10	193.384		
2	.281	10	248.737		
3	.406	10	359.442		
4	.531	10	470.147		
5	.656	10	580.852		

C ca

Step 2: Next, students will attempt to find an equation relating *C*, *A*, and *d* for a capacitor. They should move to page 1.15, which shows data on area (**ar**) and plate separation (**di**) for a capacitor. They should use the simulation on page 1.13 to find and record the capacitance for the values of *d* and *A* in the *Lists & Spreadsheet* application. Students should record these values in the column labeled **ca**.

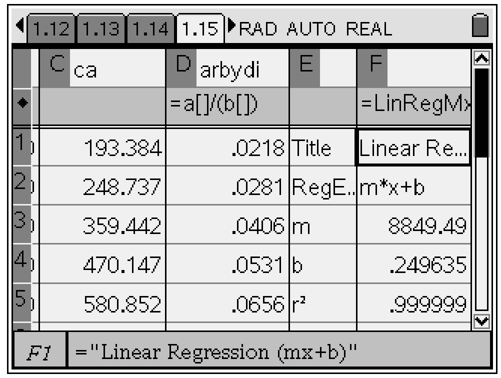
Step 3: Next, students should use column D to calculate the value of $\frac{A}{d}$ for the data in the spreadsheet. To do this, they should enter **=a[]**/**b[]** in the formula bar (light gray box) of column D. Then, students should use the **Linear Regression** tool (**Menu > Statistics > Stat Calculations > Linear Regression (mx + b)**) to find the equation relating **ca** and **arbydi**. Students should use **arbydi** in the X list and **ca** in the Y list for this calculation. Then, students should answer questions 9–12. Note: Make sure students read the text on page 1.18 before attempting to answer questions 11 and 12.



1.12 1.13 1.14 1.15 RAD AUTO REAL

	A ar	B di	C ca	D arbydi	E
◆				=a[]/(b[])	
1	.218	10	193.384	.0218	
2	.281	10	248.737	.0281	
3	.406	10	359.442	.0406	
4	.531	10	470.147	.0531	
5	.656	10	580.852	.0656	

D1 =.0218



1.12 1.13 1.14 1.15 RAD AUTO REAL

	C ca	D arbydi	E	F
◆		=a[]/(b[])		=LinRegMx
1	193.384	.0218	Title	Linear Re...
2	248.737	.0281	RegE..	m*x+b
3	359.442	.0406	m	8849.49
4	470.147	.0531	b	.249635
5	580.852	.0656	r²	.999999

F1 ="Linear Regression (mx+b)"

Q9. Write a general equation for *C* as a function of *A* and *d*.

A. *The equation is* $C = k\frac{A}{d}$, *where* k *is a constant. Students may struggle to relate the equation calculated by the* ***Linear Regression*** *tool to the equation above. Remind them that one of the variables they used for the linear regression,* ***arbydi,*** *is itself the ratio of area to plate separation.*

Q10. The constant of proportionality in the equation for *C* is the permittivity of a vacuum, ε_0. What does your linear regression predict for the value of ε_0 in units of farads per meter?

A. $\varepsilon_0 = 8.85 \times 10^{-12}$ *F/m; remind students that the plate separation,* ***di,*** *was held constant for the data they used in the linear regression. Because the plate separation was constant, students can "remove" it from the value of* k *calculated in the linear regression. This yields the value of* ε_0. *Remind students to check their units (the units for* A, d, *and* C *are given on page 1.13). Note that* ε_0 *is also known as the permittivity of free space.*

Q11. A sliver of mica is placed between the plates of a capacitor with d = 1.2 x 10^{-8} m and A = 0.050 m^2. The capacitance, C, is 260 μF. What is the dielectric constant for mica?

A. *The capacitance of a capacitor that contains a dielectric is given by the following equation:*

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Rearranging this equation to solve for ε_r, *the dielectric constant, yields the following:*

$$\varepsilon_r = \frac{Cd}{\varepsilon_0 A}$$

Substituting the given values yields the following:

$$\varepsilon_r = \frac{(260 \times 10^{-6} \ \not{F})(1.2 \times 10^{-8} \ \not{m})}{(8.85 \times 10^{-12} \ \not{F}/\not{m})(0.050 \ \not{m}^2)} = 7.1$$

Q12. What would be the capacitance without the mica?

A. *Without the mica, the capacitance decreases by a factor equal to mica's dielectric constant. In other words, the capacitance is equal to*

$\frac{260 \ \mu F}{7.1} = 37 \ \mu F = 3.7 \times 10^{-5} \ F$. *Students can confirm this calculation using the equation for the capacitance of a capacitor, as shown below:*

$$C = \frac{\varepsilon_0 A}{d}$$

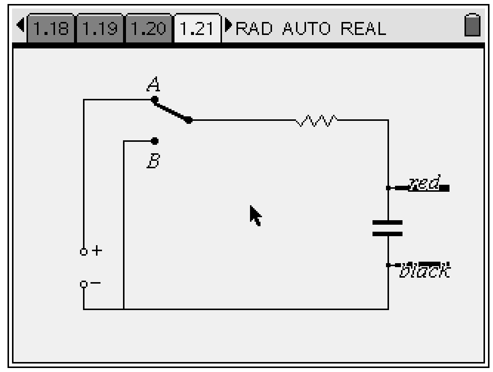
Substituting the given values and solving yields the following:

$$C = \frac{(8.85 \times 10^{-12} F/\not{m})(0.050 \ \not{m}^2)}{(1.2 \times 10^{-8} \ \not{m})}$$

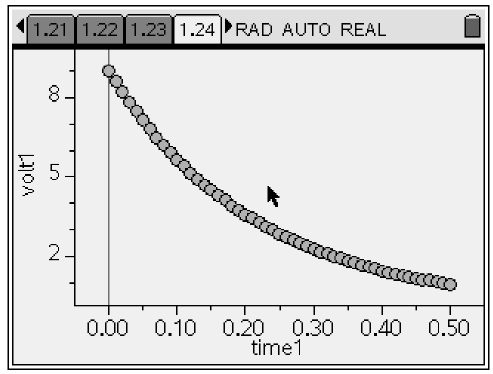
$$= 3.7 \times 10^{-5} \ F$$

You may wish to emphasize to students that calculating dielectric constants makes calculating the capacitance of the capacitor much simpler.

Part 3 – Rate of capacitor discharge



Step 1: Next, students should read the information on page 1.20, which describes the discharge from an *RC* circuit. Then, students should move to page 1.21 and examine the diagram of the *RC* circuit shown there. They should read the information on page 1.22 and then move to the *Lists & Spreadsheet* application on page 1.23, which contains experimental data for the discharge of a 10 μF capacitor in a circuit with *R* = 22 kΩ.



Step 2: Students should use the data on page 1.23 to make a graph of **volt1** vs. **time1** in the *Data & Statistics* application on page 1.24. They should use this graph to answer questions 13–15.

Q13. Describe the shape of the graph of **volt1** vs. **time1** for a discharging capacitor.

A. *Voltage decreases with time. The relationship is nonlinear and appears to follow an exponential curve. That is, the rate of change of voltage appears to decrease over time according to an exponential curve.*

Q14. How long does it take for the voltage to drop to one-half of the initial voltage?

A. *about 0.15 sec*

Q15. Show that the units of *R* • *C* (ohms • farads) are seconds.

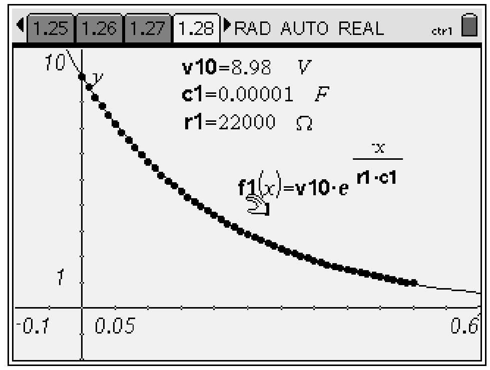
A.

$$1\ \Omega = 1\ \frac{V}{\frac{C}{s}}$$

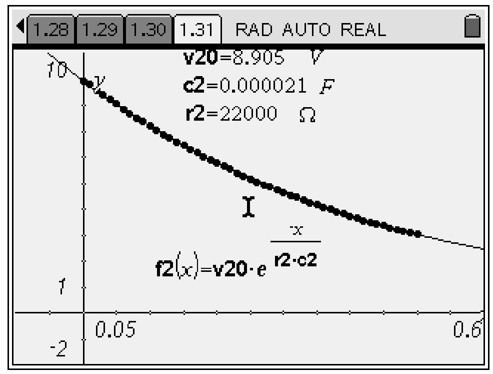
$$1\ F = 1\ \frac{C}{V}$$

$$\Omega \bullet F = \left(\frac{V}{\frac{C}{s}}\right)\left(\frac{C}{V}\right) = \left(\frac{\not{V} \bullet s}{\not{C}}\right)\left(\frac{\not{C}}{\not{V}}\right) = s$$

Step 3: Next, students should read the text on page 1.27 and then move to page 1.28. Page 1.28 shows a graph of voltage as a function of time together with data on voltage vs. time. The equation for the change of voltage with time is $V(t) = V(0)e^{(-\frac{t}{RC})}$. Students should study the fit between the equation and the data.



Step 4: Next, students should examine the data on page 1.30 and the graph on page 1.31. The data on page 1.30 make up another set of experimental data for the discharge of a capacitor. Students should change the value of **c2** until the equation fits the data well. Then, they should answer question 16.

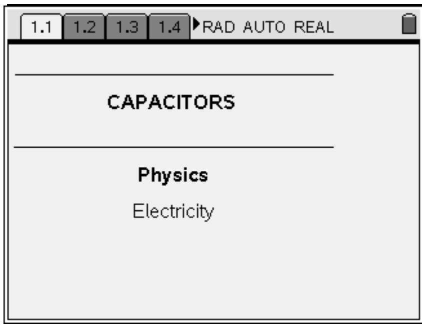


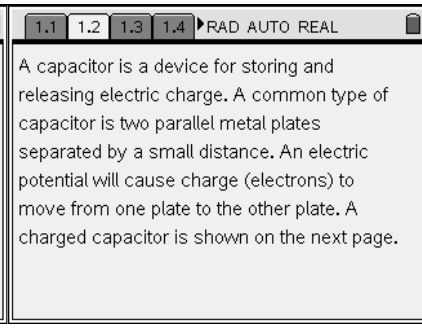
Q16. What is the capacitance of this second capacitor?

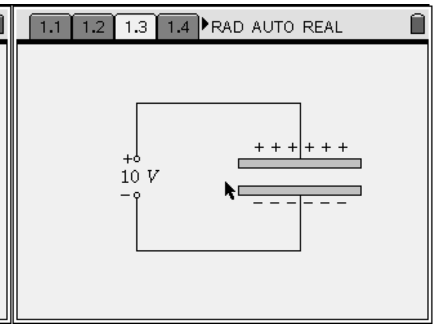
A. *21 μF*

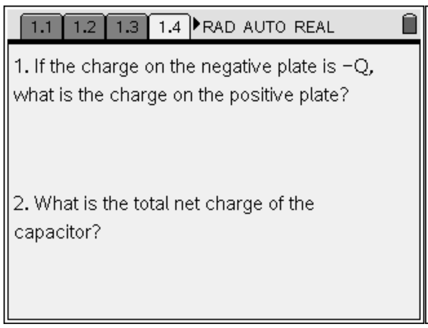
Capacitors – ID: 9460

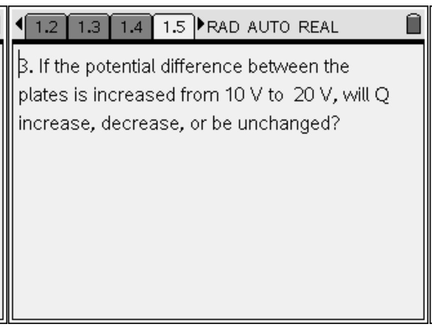
(Student)TI-Nspire File: *PhyAct16_capacitors_EN.tns*

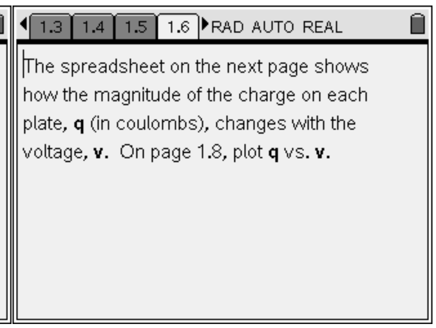


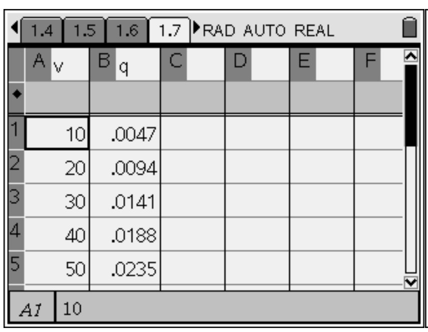


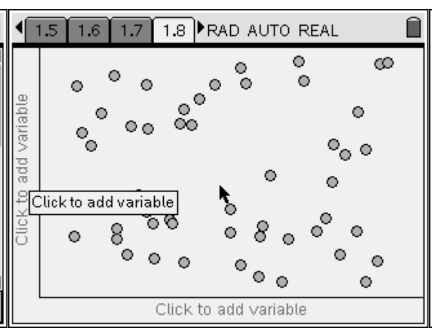


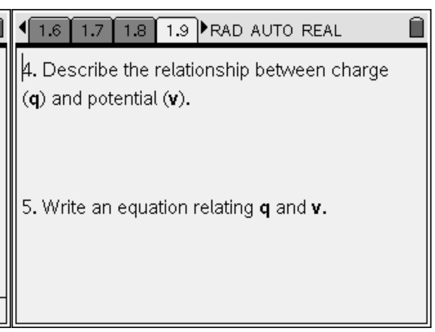


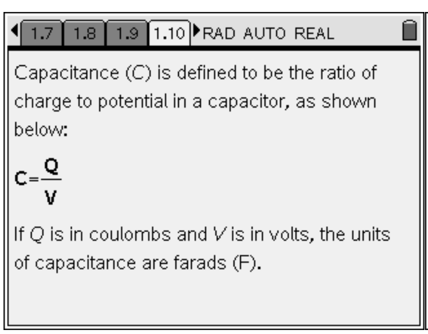


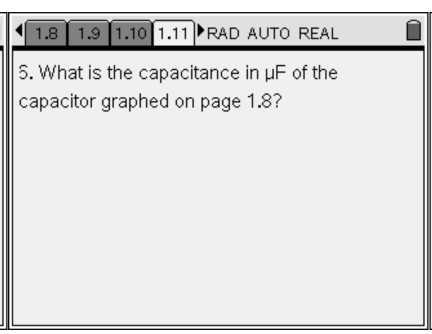


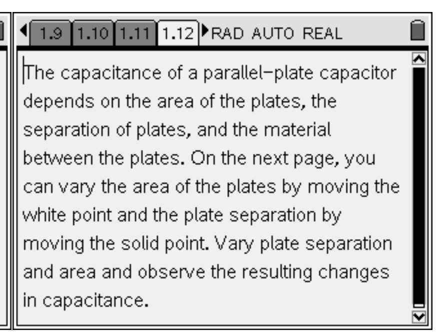


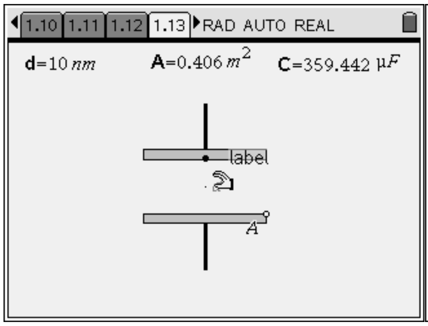












1.11 1.12 1.13 1.14 RAD AUTO REAL

7. How does increasing the plate area affect capacitance?

8. How does decreasing plate separation affect capacitance?

1.12 1.13 1.14 1.15 RAD AUTO REAL

	A ar	B di	C ca	D arbydi	E
◆					
1	.218	10			
2	.281	10			
3	.406	10			
4	.531	10			
5	.656	10			

A1 .218

1.13 1.14 1.15 1.16 RAD AUTO REAL

9. Write a general equation for *C* as a function of *A* and *d*.

1.14 1.15 1.16 1.17 RAD AUTO REAL

10. The constant of proportionality in the equation for *C* is the permittivity of a vacuum, ε_0. What does your linear regression predict for the value of ε_0 in units of farads per meter?

1.15 1.16 1.17 1.18 RAD AUTO REAL

This equation for capacitance assumes the space between the plates is a vacuum (or air). In a typical capacitor, the space is filled with a polar insulator known as a *dielectric*. Adding a dielectric multiplies the capacitance by a factor equal to the dielectric constant, ε_r.

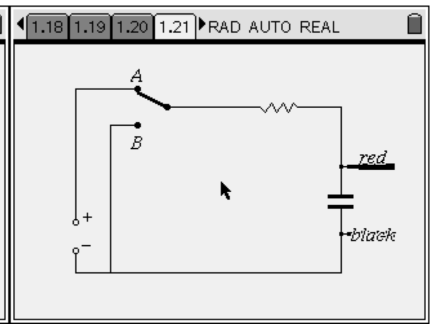
1.16 1.17 1.18 1.19 RAD AUTO REAL

11. A sliver of mica is placed between the plates of a capacitor with $d = 1.2 \times 10^{-8}$ m and $A = 0.050$ m^2. The capacitance, *C*, is 260 µF. What is the dielectric constant for mica?

12. What would be the capacitance without the mica?

1.17 1.18 1.19 1.20 RAD AUTO REAL

One of the features of capacitors is the ability to discharge very rapidly. The schematic on the next page shows an experimental setup for measuring the rate of discharge of a capacitor. With the switch at point *A*, the capacitor is charged. Moving the switch to point *B* discharges the capacitor. The red and black probes measure the voltage.



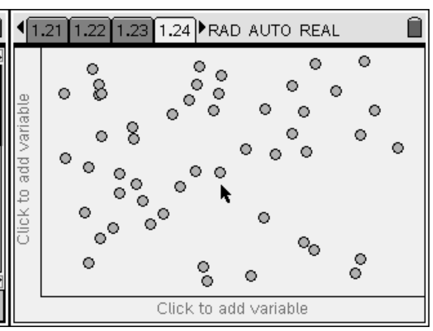
1.19 1.20 1.21 1.22 RAD AUTO REAL

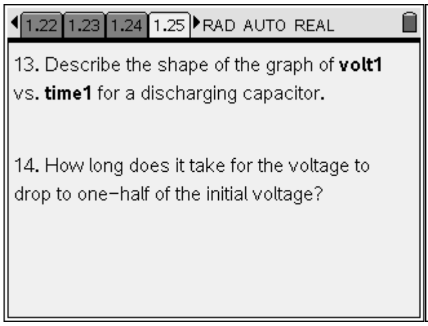
The data on the next page were collected for this circuit with *R* = 22 kΩ and *C* = 10 µF. The voltage probes were connected to a CBL data collection unit attached to a TI-84 calculator. The units of time are seconds. Make a plot of **volt1** vs. **time1** on page 1.24.

1.20 1.21 1.22 1.23 RAD AUTO REAL

	A time1	B volt1	C	D	E	F
◆						
1	0	8.97924				
2	.01	8.58852				
3	.02	8.1978				
4	.03	7.82662				
5	.04	7.47009				

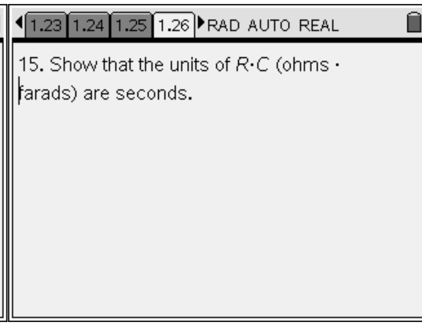
A1 0



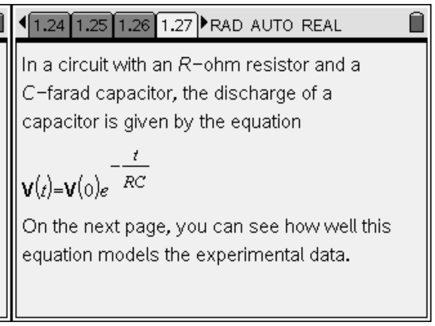

1.22 1.23 1.24 1.25 RAD AUTO REAL

13. Describe the shape of the graph of **volt1** vs. **time1** for a discharging capacitor.

14. How long does it take for the voltage to drop to one−half of the initial voltage?


1.23 1.24 1.25 1.26 RAD AUTO REAL

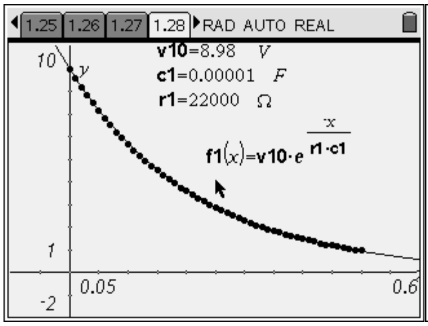
15. Show that the units of $R \cdot C$ (ohms · farads) are seconds.

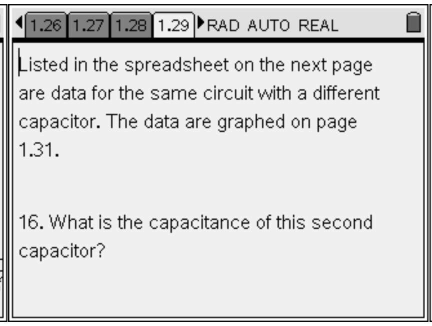

1.24 1.25 1.26 1.27 RAD AUTO REAL

In a circuit with an R−ohm resistor and a C−farad capacitor, the discharge of a capacitor is given by the equation

$$v(t)=v(0)e^{-\frac{t}{RC}}$$

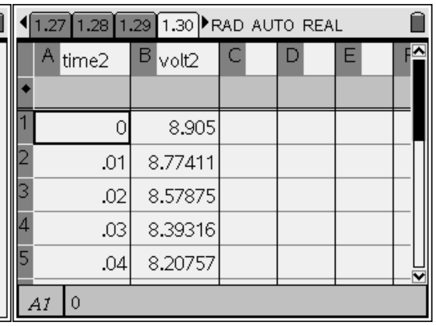
On the next page, you can see how well this equation models the experimental data.




1.26 1.27 1.28 1.29 RAD AUTO REAL

Listed in the spreadsheet on the next page are data for the same circuit with a different capacitor. The data are graphed on page 1.31.

16. What is the capacitance of this second capacitor?


1.27 1.28 1.29 1.30 RAD AUTO REAL

	A time2	B volt2	C	D	E
◆					
1	0	8.905			
2	.01	8.77411			
3	.02	8.57875			
4	.03	8.39316			
5	.04	8.20757			

A1 0