

NUMB3RS Activity: Sharpshooter Episode: "Waste Not"

Topic: Geometric probability

Grade Level: 9 - 10

Objective: Use geometric probability to calculate the likelihood of an event. Determine whether it is appropriate to make conclusions based on an "after the fact" situation.

Materials: TI-83 Plus/TI-84 Plus graphing calculator

Time: 15 - 20 minutes

Introduction

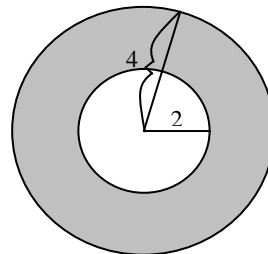
In "Waste Not," the FBI discovers that children in the area of a sinkhole have what seems to be an unusually high occurrence of cancer. FBI agent Megan Reeves believes that this might represent a "cancer cluster." Charlie warns her not to jump to that conclusion too quickly. He says that cancer clusters are statistically rare and that this is more likely to be a case of "Texas Sharpshooting." He explains this concept by using an example of a man who has never held a gun before. The man shoots several times at the side of a barn. He then walks up to the barn, draws a circle around those holes that are closest to each other. Because they fit in this "cluster," he declares himself to be a Texas Sharpshooter.

In this activity, students investigate and simulate Texas Sharpshooting.

Discuss with Students

This activity is designed for students to explore the use of geometric probability. If something rare occurs, is it the result of skill or luck?

Introduce geometric probability with this example: Given two concentric circles with radii 2 units and 4 units, as shown at the right. The area of the smaller circle is 4π square units and the area of the larger circle is 16π square units. The area of the shaded region is 12π square units, or three times the area of the smaller circle. Therefore the probability of a random dart landing in the shaded region is three times the probability of landing inside the small circle.



In the calculator program SHARP, the first six lines of the program initialize the calculator and set the window to $x: [-15.16, 15.16]$, $y: [-10, 10]$. With this window, the graph of a circle will look like a circle. The last five lines of the program contain a loop that calculates and plots ten shots. The coordinates of each shot are determined by random integer commands. The **Text** command plots each shot. If you have access to a TI-Navigator™ system, you may want to use it to display the students' screens as they run the program.

Note that while some students may find it beneficial to actually key-in the SHARP program on their calculators, the program file can also be downloaded for free by going to <http://education.ti.com/exchange> and searching for "7687."

Time and space permitting, consider modeling this activity by having students close their eyes and toss chalky blackboard erasers at a wall.

Student Page Answers:

1. 606.4 ft^2
2. Radius = 8.80 feet; Area = $77.46\pi \approx 243.36 \text{ ft}^2$
3. 0.40
4. 0.40
5. $(0.40)^{10} \approx 0.00011$
6. Some students may conclude that the shots were not random, since the probability of all 10 shots landing in this particular circle is so small. Others might be suspicious because the circle was drawn after the shots occurred. Guide discussion so that they realize that it is inappropriate to use probability for events after they have occurred. In fact, there is no way to look at the pattern of shots and determine whether they were random or not.

Extensions Answers:

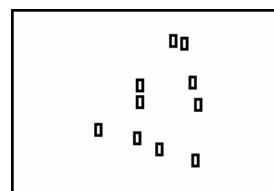
1. Possible solution: change the lines in the program to $\text{randInt}(20, 30) \rightarrow X$ and $\text{randInt}(25, 55) \rightarrow Y$
2. Possible solution: in the original program, change the line $\text{randInt}(0, 80) \rightarrow Y$ to $\text{randInt}(0, X) \rightarrow Y$
3. Change the line $\text{For}(B, 1, 10)$ to $\text{For}(B, 1, 20)$. Increasing the number of shots may increase the likelihood of seeing clusters that are a result of chance. Not all 20 points will necessarily be within the same cluster, and with more shots, it may be more likely to have more than one cluster.

Name: _____ Date: _____

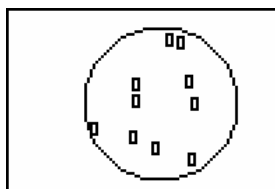
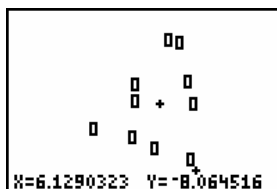
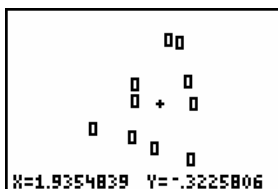
NUMB3RS Activity: Sharpshooter

In "Waste Not," the FBI discovers that children in the area of a sinkhole seem to have an unusually high occurrence of cancer. FBI agent Megan Reeves believes that this might represent a "cancer cluster." Charlie warns her not to jump to that conclusion too quickly. He says that cancer clusters are statistically rare and that this is more likely to be a case of "Texas Sharpshooting." He explains this concept by using an example of a man who has never held a gun before. The man shoots several times at the side of a barn. He then walks up to the barn and draws a circle around those holes that are closest to each other. Because they fit in this "cluster," he declares himself to be a Texas Sharpshooter.

- Imagine that the picture at right represents the side of a barn. The width of the barn is 30.32 feet and its height is 20 feet. What is the area of the side of the barn?



- Suppose the middle of the side of the barn is the origin for a graph. The first figure below shows that the center of a circle could be located approximately at $(1.94, -0.32)$; the second figure shows one possible point on the circle approximately at $(6.13, -8.06)$; and the third figure shows a circle drawn around the 10 points. Based on this information, what is the radius of the circle? What is the area of the circle?



- What is the ratio of the area of the circle to the area of the side of the barn?
- If the shots are fired at random, what is the probability that a shot lands inside this circle?
- If the shots are fired at random, what is the probability that all 10 shots land inside the circle?
- Does the answer to Question #5 convince you whether or not the shots were likely fired at random?

Try this simulation yourself, by entering the following program in your calculator. The individual commands can be found in the calculator's Catalog (press $\boxed{2nd}$ [CATALOG]).

This program first sets the calculator's window to $x: [-15.16, 15.16]$, $y: [-10, 10]$. Then the program picks a random x -coordinate (from 0 to 50) and a random y -coordinate (from 0 to 80) for the location of each shot.

```
PROGRAM: SHARP
:FnOff
:PlotsOff
:ZStandard
:ZSquare
:ClrDraw
:AxesOff
:For(B,1,10)
:randInt(0,50)+X
:randInt(0,80)+Y
:Text(X,Y,"0")
:End
```

Run the program several times until you see a cluster of shots, and then draw a circle around the cluster. To do this, press \boxed{PRGM} , and select the program **SHARP**. Then press \boxed{ENTER} twice. If necessary, run the program again to get a cluster. Then, while on the graph screen, press $\boxed{2nd}$ [DRAW] and choose **9:Circle(**. Use the arrow keys to move the cursor to the center of your cluster. Press \boxed{ENTER} . Record the coordinates of the center. Move the cursor to choose a point on the circle and press \boxed{ENTER} to draw the circle. Record the coordinates of the point on the circle and compute both the radius and area of the circle.

```
9:Circle(
:Horizontal
:Vertical
:Shade(
:DrawF
:DrawInv
:Circle(
```

Your goal is to get all of your shots into a circle that fits on the calculator's screen. Find the area of the circle. Then find the probability that you could have achieved this same result if the circle had been specified ahead of time. If the probability is less than 0.001, then you are a Texas Sharpshooter! Compare your results with the results of your classmates.

The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

The calculator program SHARP is designed to simulate 10 shots fired at random. By editing different parts of the program, it can be made to simulate different scenarios.

1. Edit the calculator program so that the shots always land closer to the middle of the screen.
2. Imagine that a diagonal is drawn on the screen. Edit the calculator program so that the shots always land on one side of the diagonal.
3. Edit the calculator program so that 20 shots are taken each time. Does increasing the number of shots increase the likelihood of becoming a Texas Sharpshooter?

Additional Resources

For an extensive annotated list of links to material on randomness and geometric probability, including Buffon's needle, random domino tilings, and random polygons, see Randomness and Geometric Probability (The Geometry Junkyard) – David Eppstein, Theory Group, ICS, UC Irvine, at: <http://www.ics.uci.edu/~epstein/junkyard/random.html>

For more calculator programs, go to these Web sites:

<http://education.ti.com/educationportal/sites/US/nonProductMulti/programarchive.html>

<http://www.ticalc.org/>