Name ______________________

Class ______________________

Problem 1 – Investigating the Graph of $y = e^x$

On page 1.4, the graph of **f1**(x) = e^x is shown along with the tangent to **f1**(x).

1. Explain how the shaded triangle has been constructed. What is the base and height?

2. Find the slope of the tangent at the point (a, $f(a)$). Show your work.

3. Drag the point on the x-axis and observe the effect on the triangle. Describe what you notice about the triangle. What stays the same? What changes? Why do you think this happens?

4. Based on your observations, what is the least amount of information that will allow you to determine the slope of the tangent line?

5. How does slope of the tangent line relate to the function $y = e^x$?

6. Summarize your findings by writing the derivative of $y = e^x$.

7. To confirm your observations, advance to page 1.8 to set up and evaluate $\lim\limits_{h\to 0}\dfrac{e^{a+h}-e^a}{h}$.

   Based on this result, what is the value of the derivative of $y = e^x$ at $x = a$?

8. Now set up and evaluate an expression that represents the derivative, as a function of x, for the function $y = e^x$.

Problem 2 – Generalizing Your Findings

Using the definition of the derivative, the derivative of $y = e^x$ is defined as

$$\frac{d}{dx}\left(e^x\right) = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}.$$

Using a variety of properties from algebra, the right side of this equation can be simplified as the following:

$$\begin{aligned}\frac{d}{dx}\left(e^x\right) &= \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}\\ &= \lim_{h\to 0}\frac{e^x\left(e^h-1\right)}{h}\\ &= e^x\cdot\lim_{h\to 0}\frac{e^h-1}{h}.\end{aligned}$$

In the first part of this activity, you made a conjecture as to the derivative of $y = e^x$.

9. What must the value of $\lim_{h\to 0}\frac{e^h-1}{h}$ be equal to for this conjecture to be true?

Advance to page 2.2, where you will see a graph of $f(x)=\frac{e^x-1}{x}$ and its function table. Drag the point on the *x*-axis towards the origin and observe the value of this limit graphically.

10. What does the limit as x approaches 0 appear to be? What is the value of $\lim_{h\to 0}\frac{e^h-1}{h}$?

The derivative of $y = e^x$ can be extended to any exponential function, $y = a^x$ where $a > 0$ and $a \neq 0$. Go to page 2.4 and set up and evaluate the limit $\lim_{h\to 0}\frac{a^{x+h}-a^x}{h}$.

11. What is the derivative of $y = a^x$.

12. The chain rule can also be used to find the derivative of $y = a^x$. Rewrite $y = a^x$ with base e, using the inverse property of exponents and logarithms, $e^{\ln x} = x$. Using the chain rule and the fact that $\frac{d}{dx}\left(e^x\right)=e^x$, confirm the result you found on the calculator screen. Then evaluate $\frac{d}{dx}\left(a^x\right)$ using the **derivative** template in the calculator application. Did you get the same result?