Absolute Value Functions

Concepts	Overview
• The general form of an absolute value function is $f(x) = a|x - h| + k$. By changing the values of a, h, and k, the shape and position of the graph will be modified.	Transformation of graphs **Materials** TI84+ graphing calculator, graph paper, ruler.

Set-up of values from the home screen and the equations in the $\boxed{Y=}$ screen.

- From the home screen, store values for a, h, and k. $\boxed{1}\boxed{STO\blacktriangleright}\boxed{ALPHA}$ A $\boxed{ALPHA}$ $\boxed{.}$ $\boxed{0}$ $\boxed{STO\blacktriangleright}\boxed{ALPHA}$ H.$\boxed{ALPHA}$ $\boxed{.}$ $\boxed{0}$ $\boxed{STO\blacktriangleright}\boxed{ALPHA}$ K $\boxed{ENTER}$. See Figure 1. Note: You'll be returning to this screen to change the values of a, h, and k throughout this exploration.
- From the $\boxed{Y=}$ enter the reference function and the general function of an absolute value function. $\boxed{Y=}\boxed{MATH}\boxed{\blacktriangleright}\boxed{1}\boxed{X,T,\Theta,n}\boxed{)}\boxed{ENTER}$A $\boxed{MATH}\boxed{\blacktriangleright}\boxed{1}\boxed{X,T,\Theta,n}\boxed{-}H\boxed{)}\boxed{+}$K. Note: The first function is the **reference function** $y = |x|$ and the second function is the **general form of the absolute value function** $f(x) = a|x - h| + k$.
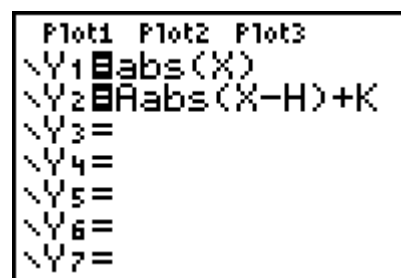
Graph the functions.

- Press $\boxed{ZOOM}\boxed{6}$ for a standard graphing window. See Figure 3. Note: The scales along each axis are not equal. Also, the graphs coexist.
- Press $\boxed{ZOOM}\boxed{8}\boxed{ENTER}$ for the integer graph window. See Figure 4. Note: The scales on each axis are equal. Complete the table for these functions.
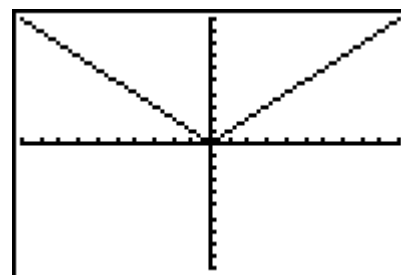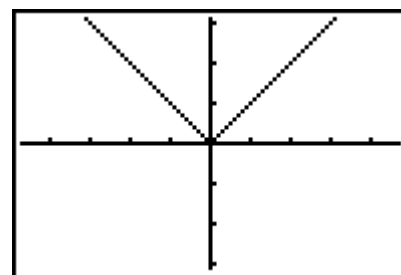
x	-3	-1	0	2	4
y					

Figure 1

Figure 2

Figure 3

Figure 4

Moving the graph vertically.
- Return to the home screen 2nd MODE.
- Press 2nd ENTER. Use the arrow keys and change the value for K to 5. See Figure 5. Be sure to press ENTER.

The general form of the absolute value function has been changed to $f(x)=|x|+5$
- Return to the graph window by pressing GRAPH. See Figure 6.
- Use the TRACE feature and complete the table below for both the reference function $y=|x|$ and the general form of the absolute value function when K = 5.

x	−5	−2	0	1	4
$y=|x|$					
$y=|x|+5$					

What is the vertex? The vertex of the absolute value function is point where the graph changes direction. In the above example the vertex $y=|x|$ is $(0,0)$ and the vertex of the function $f(x)=|x|+5$ is $(0,5)$.

You can return to the home screen and change the K value (see Figure 5) and then return to the graph (see Figure 6). Press TRACE − 5 ENTER ▾ to get Figure 7. Note: The curser is at the vertex of $f(x)=|x|+5$.

Complete the table for each value of K by writing the general form of the absolute value function in simplified form and find the vertex. The first one is done for you.

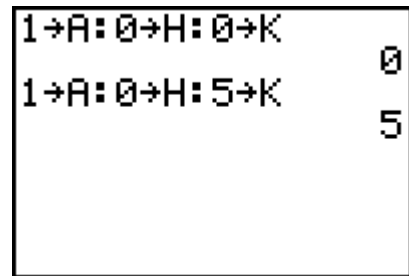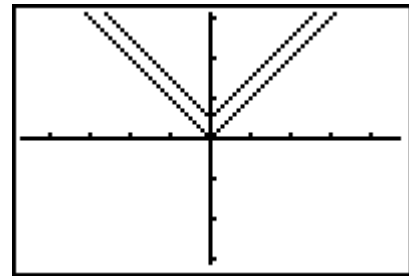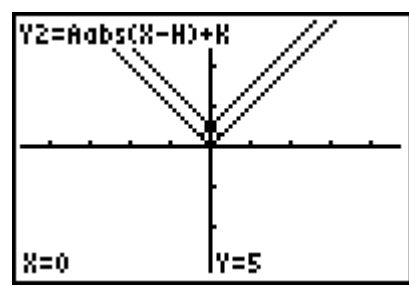| k	Function	Vertex
15	$y=|x|+15$	$(0, 15)$
−10		
25		
−30		

Figure 5

Figure 6

Figure 7

Generalize your findings about the effect of K on the graph of $f(x)=a|x-h|+k$ with respect to the reference graph $f(x)=|x|$.

Moving the graph horizontally.
- Return to the home screen [2nd][MODE].
Press [2nd][ENTER]. Use the arrow keys and change the value for H to 10. See Figure 8. Be sure to press [ENTER].

The general absolute value has been changed to $f(x)=|x-10|$
- Return to the graph window by pressing [GRAPH]. See Figure 9.

Use the [TRACE] feature and complete the table below for both the reference function $y=|x|$ and the general form of the absolute value function when H = 10

x	−6	−3	0	2	5
$y=|x|$					
$y=|x-10|$					

Complete the table for each of the values of H. See Figure 10 for vertex of the first function. Where is the vertex?

h	Function	Vertex
10	$y=|x-10|$	$(10,\ 0)$
−15		
25		
−30		

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1→A:0→H:0→K
                    0
1→A:0→H:5→K
                    5
1→A:10→H:0→K
                    0
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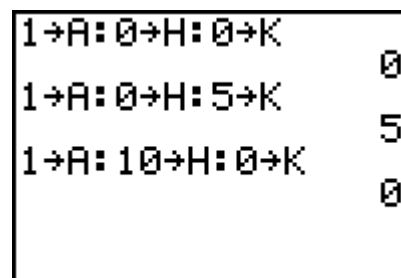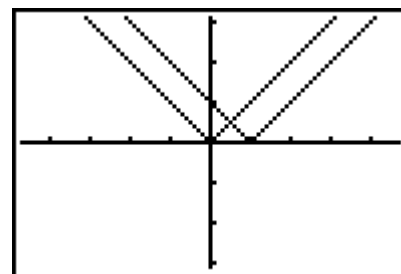Figure 8

Figure 9

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Y2=Aabs(X-H)+K
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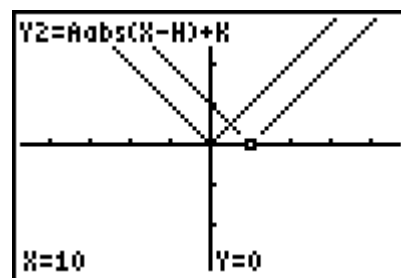```
X=10   Y=0
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Figure 10

Generalize your findings about the effect of H on the graph of $f(x) = a|x-h| + k$ with respect to the reference graph $f(x) = |x|$.

What are the slopes of the two sides of this graph?

Return to the GRAPH screen. Determine the slopes of the left and right "sides" of these graphs. Figure 11 is the same as Figure 10.

Slope of the graph left of the vertex =

Slope of the graph right of the vertex =


Figure 11

**Changing the slopes and direction of opening**.

Return to the home screen 2nd MODE and change the values as seen in Figure 12.

Return to the GRAPH screen and TRACE ▼ 1 0 ENTER to get Figure 13. The general form of the equation of the absolute value has been changed to $y = 2|x|$.

Slope of the graph left of the vertex =

Slope of the graph right if the vertex =

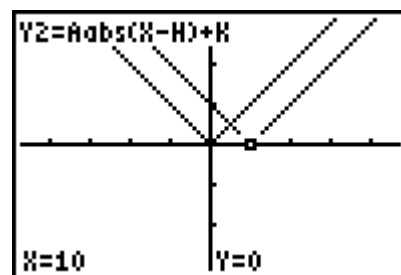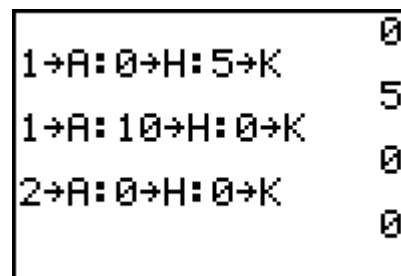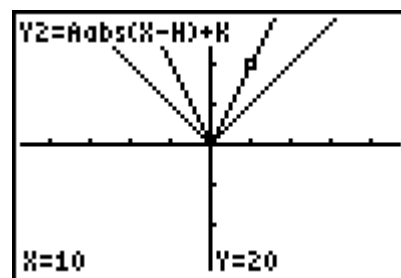Note: The direction of opening is unchanged. It is still up.


Figure 12


Figure 13

Using the home screen, GRAPH, and TRACE complete the
following table for each value of A.

Where is the vertex for each of these graphs?

| $A$ | Function | Direction of Opening | Slope left of vertex | Slope right of vertex |
|---|---|---|---|---|
| 2 | $y = 2|x|$ | Up | | |
| 3 | | | | |
| $\dfrac{1}{2}$ | | | | |
| $\dfrac{1}{4}$ | | | | |

Using the home screen, GRAPH, and TRACE complete the
following table for each value of A.

Where is the vertex for each of these graphs?

| $A$ | Function | Direction of Opening | Slope left of vertex | Slope right of vertex |
|---|---|---|---|---|
| $-2$ | $y = -2|x|$ | Down | | |
| $-3$ | | | | |
| $-\dfrac{1}{2}$ | | | | |
| $-\dfrac{1}{4}$ | | | | |

Generalize your findings about the effect of A on the graph of $f(x)=a|x-h|+k$ with respect to the reference graph $f(x)=|x|$.

Return to the home screen 2nd MODE and change the values as seen in Figure 14.

Return to the GRAPH screen and TRACE ▼ 1 0 ENTER to get screen 15. The general form of the equation of the absolute value has been changed to $y=-|x+3|-2$.

Answer each of the following questions for the graph of $y=-|x+3|-2$.

What are the coordinates of the vertex?

What is the direction of opening?

Slope of graph left of the vertex =

Slope of graph right of the vertex =

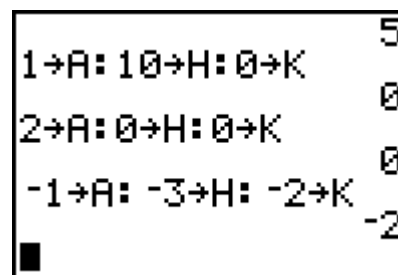What is the equation of the graph given in Figure 16?

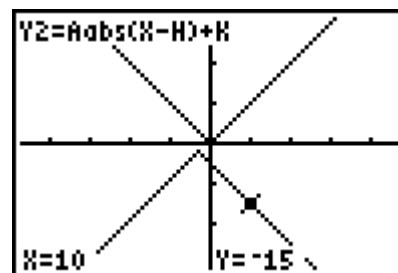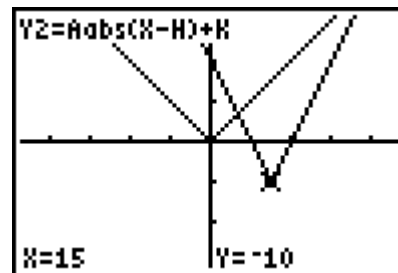What are the values for

A =

H =

K =


Figure 14


Figure 15


Figure 16