EXPLORATIONS

Activity 3

Objective

- To use Geoboard to find the areas of triangles having the same base and the same height

Materials

- TI-73
- Student Activity pages (pp. 23 – 30)

Move Along Little Vertex

In this activity you will

- Recognize that the area of a triangle depends on the length of its base and its height.
- Discover that a triangle's area is one-half the product of its base and height.

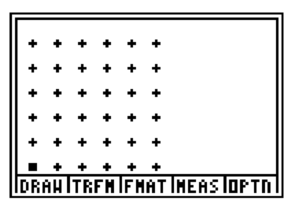
Introduction

All triangles can be changed to a right triangle that has the same area. Finding the area of a right triangle is fairly easy: we already know the base and the height, since it is half of a rectangle. In this activity, you will explore the idea of changing non-right triangles to right triangles to help make finding the area easier.

Investigation

This investigation will help you find the areas of triangles that have the same base and same height.

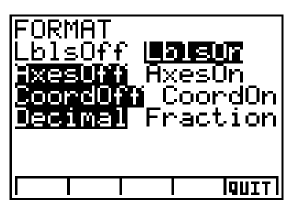
1. From the main Geoboard menu, select **2:6x6**.



2. To format the geoboard, select **FMAT** and make sure that the following settings are selected:

   LblsOn (Labels are on)
   AxesOff (Axes are off)
   CoordOff (Coordinates are off)
   Decimal (Measurement is in decimal form)

   Select **QUIT** to exit the FORMAT menu.



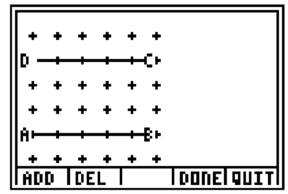
3. On this geoboard, construct two parallel line segments.

   a. Starting at the lower left peg, move the cursor up one unit by pressing ▲.

   b. To create the first line segment, select **DRAW**, **ADD** ▶ ▶ ▶ ▶ ▶ **DONE**.

   c. Move up three units by pressing ▲ ▲ ▲ and create the second line segment by selecting **ADD** ◀ ◀ ◀ ◀ ◀ **DONE**.

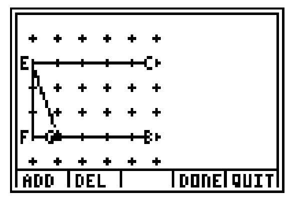
   Your screen should show two line segments AB and DC. Your geoboard should look like the screen at the right.



4. Between these two parallel lines, build three different triangles. Each triangle will have the same base and the same height.

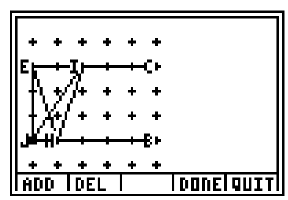
   a. For Triangle 1, start at D and select **ADD** ▼ ▼ ▼ **ADD** ▶ **DONE**.

   Your geoboard should look like the screen at the right.



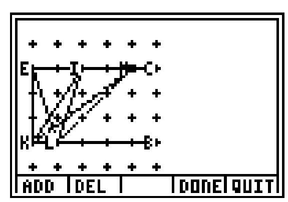
   b. For Triangle 2, start at G and select **ADD** ▶ ▲ ▲ ▲ **ADD** ◀ ◀ ▼ ▼ ▼ **DONE**.

   Your geoboard should look like the screen at the right.



   c. For Triangle 3, start at J and select **ADD** ▶ **ADD** ▶ ▶ ▶ ▲ ▲ ▲ **DONE**.

   Your geoboard should look like the screen at the right.



5. Find the area of each of the three triangles by moving the cursor to point L, which is common to all three triangles, and selecting **QUIT**, **MEAS**, **2:Area** [ENTER]. Triangle EKL is highlighted. Press [ENTER] to find the area. Select **QUIT** before repeating this step for the other two triangles. Use the arrow keys to select the other triangles and find their areas. Record the area of each triangle.

   a. Triangle 1 (EFG) ________________

   b. Triangle 2 (JHI) ________________

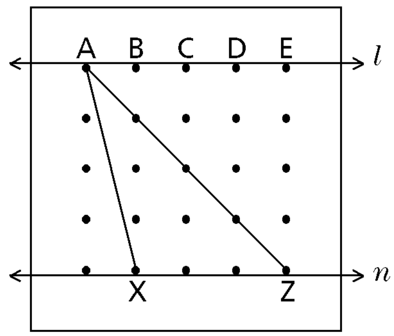
   c. Triangle 3 (KLM) ________________

Student Activity

Name ____________________________

Date ____________________________

Activity 3.1: Move Along Little Vertex



Select a 5×5 board and construct parallel lines l and n. Make five triangles with line segment XZ as the base of each triangle on your geoboard. The third vertex will be A, then B, then C, then D, and finally E. For each triangle, identify the base and determine the base length, the height, and the area. Record this information in the table below and then answer the following questions. Make sure to check your answers with the TI-73.

Triangle	Triangle Base	Length of Base	Height of Triangle	Area of Triangle
XZA	XZ	3		
XZB	XZ	3		
XZC	XZ			
XZD			4	
XZE				

As the vertex moves from A to B to C to D to E to form each new triangle with base XZ:

1. How does the height of the triangle change? Why?

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____

____

2. How does the area of the triangle change? Why?

____

____

____

3. If you were asked to find the area of triangle XZA, which “nicer” triangle can you change it into and still have the same base and area? Why?

4. Based on the information in the table, what is the minimum information you need to know to find the area of a triangle?

5. How can this information be used to determine a triangle’s area?

6. If we were going to build a triangular horse pen with the same area as triangle XZA above and part of fence XZ was already built, what triangle would you build that would use the least amount of fence but not change the area?

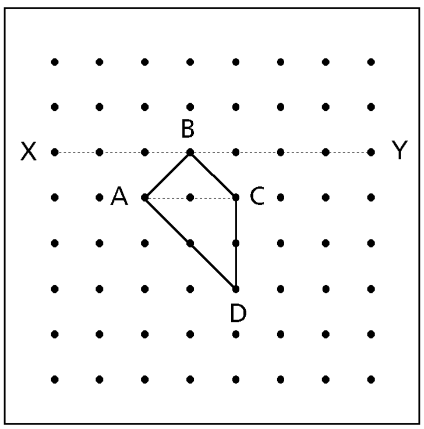
Student Activity

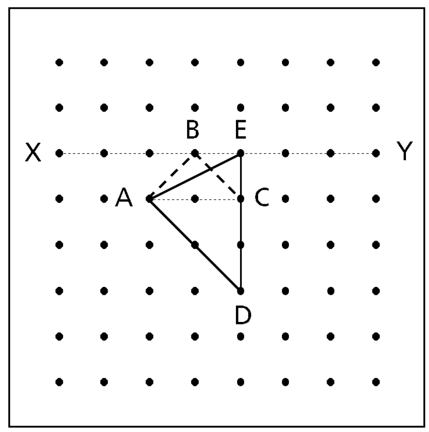
Name ____________________________

Date ____________________________

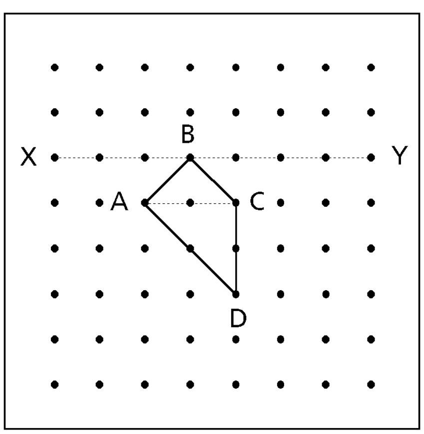
Activity 3.2: The Quadrilateral Tile Company

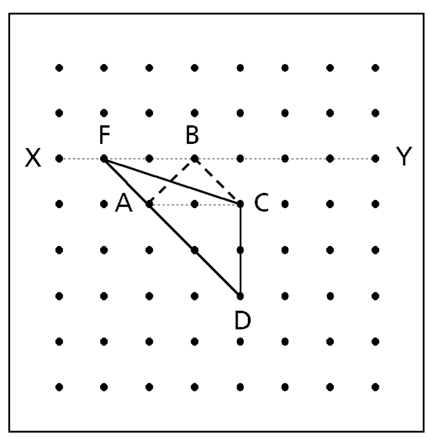
The Quadrilateral Tile Company has been making four-sided tiles for years. They would like to get more variety by making triangular tiles that have the same area as each of their quadrilateral tiles. Sytia, an employee, decides that she can use triangles between parallel lines to redesign tiles like ABCD and still keep the same area. She makes diagonal AC and then makes a segment XY through B that is parallel to diagonal AC. Now triangle ABC is between the parallel line segments AC and XY. If she moves B anywhere along segment XY, the new triangle will have the same area as triangle ABC. For example, if B moves to E, triangle ABC changes to triangle AEC and still has the same area.





Notice that the part of the shape below diagonal AC has not changed. This means that the shape AED, which is now a triangle, has exactly the same total area as quadrilateral ABCD. Use your TI-73 to check the area of quadrilateral ABCD and triangle AED to see that they are the same.



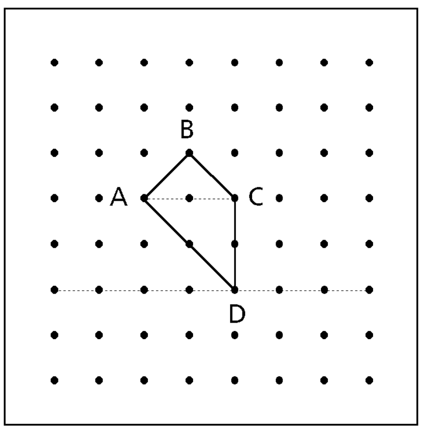


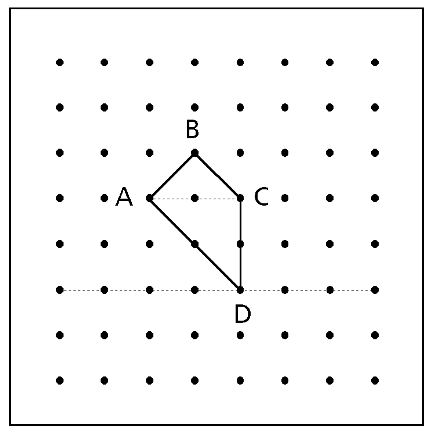
In a similar way, Sytia could have moved point B to point F, creating triangle ACF with the same area as triangle ABC. This would create a different larger triangle DFC with the same area as quadrilateral ABCD.

The Quadrilateral Tile Company can follow the steps below to redesign quadrilateral tiles into triangular tiles with the same area.

1. Establish a convenient diagonal that splits the shape into two triangles.
2. Choose either vertex not on the diagonal and construct a line segment through this point that parallels the original diagonal.
3. Move that vertex in either direction along the parallel line until the entire shape changes from a quadrilateral into a triangle.

Now use these steps with vertex D to redesign quadrilateral tile ABCD into a triangular tile with the same area. There are two possible solutions. Sketch each one below and check your answer with the TI-73.

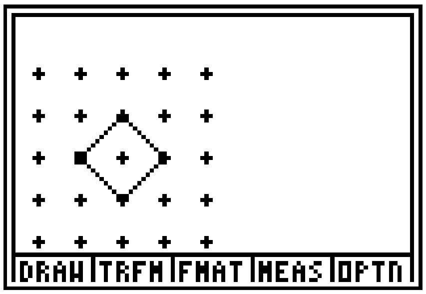


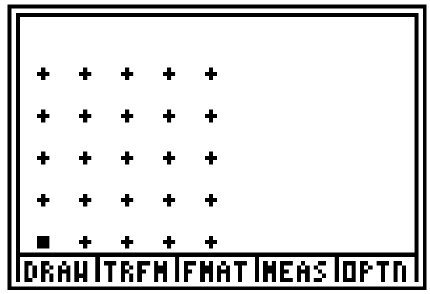


Now redesign some of this company's other tiles from quadrilaterals into triangles with the same area by using triangles between parallel lines. For each tile, move just one vertex (point) so that the tile changes from a quadrilateral into a triangle with the same area. Make a sketch of at least one possible triangular tile. Check the areas of both figures on your TI-73.

Original tile | *Your triangular design*

1.





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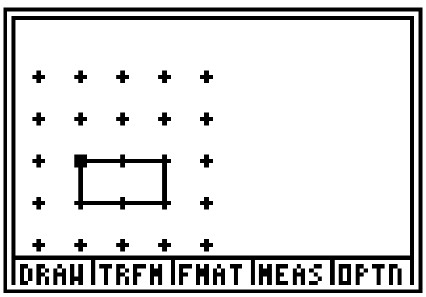

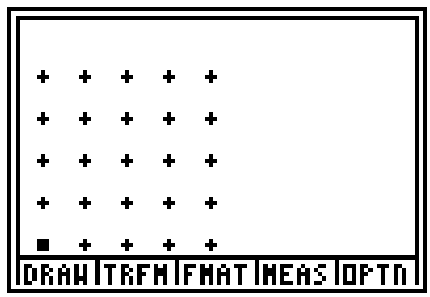

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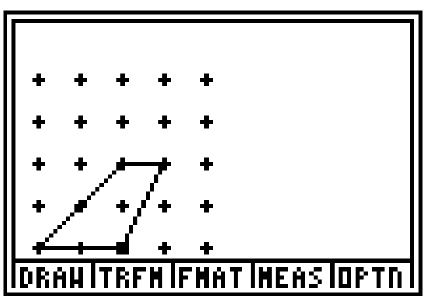

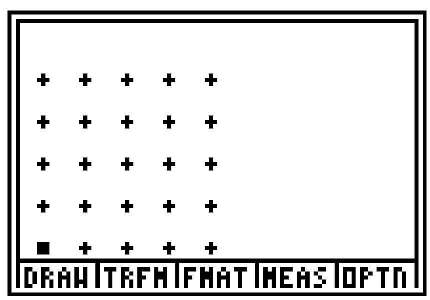

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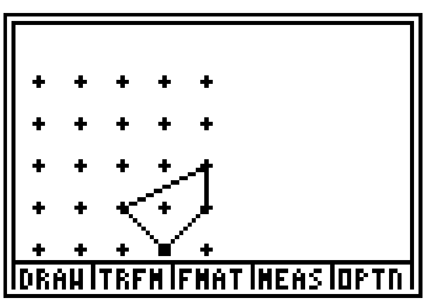
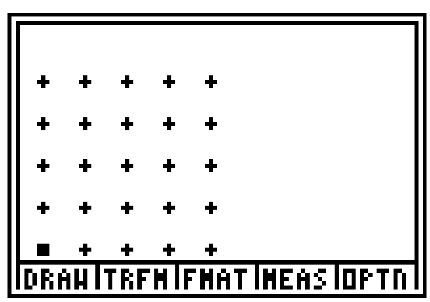




Student Activity

Name ____________________________

Date ____________________________

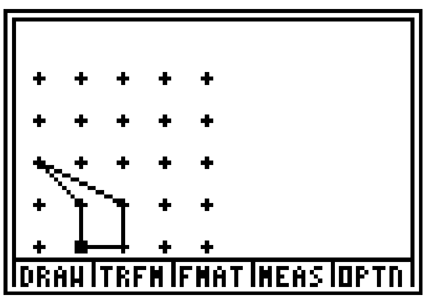
Activity 3.3: The Pentagonal Tile Company

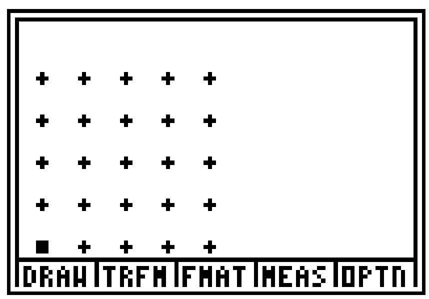
The Pentagonal Tile Company has been making five-sided tiles for years. They would now like to compete with the Quadrilateral Tile Company by adding quadrilateral tiles with the same area as the pentagonal tiles. Like the Quadrilateral Tile Company, the Pentagonal Tile Company will use triangles between parallel lines to redesign its tiles.

You are an employee of the Pentagonal Tile Company assigned the task of redesigning each pentagonal tile into a quadrilateral tile with the same area. For each tile, move just one vertex so that the tile changes from a pentagon into a quadrilateral with the same area. Draw a sketch of at least one new possible quadrilateral tile. Use your TI-73 to check that the areas are the same.

Original tile *Your quadrilateral design*

1.





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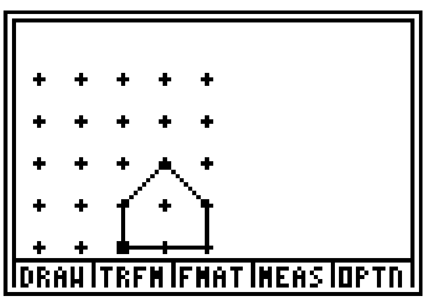

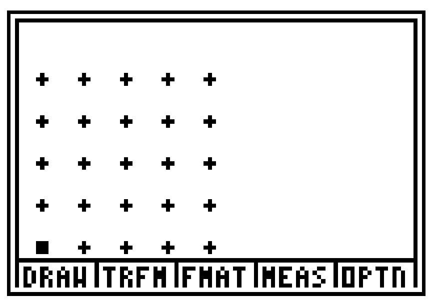

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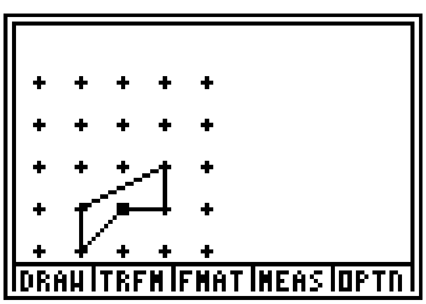

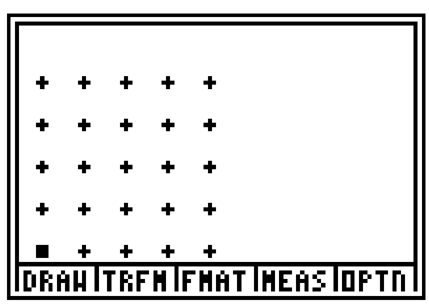

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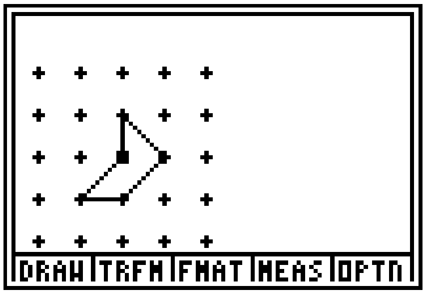
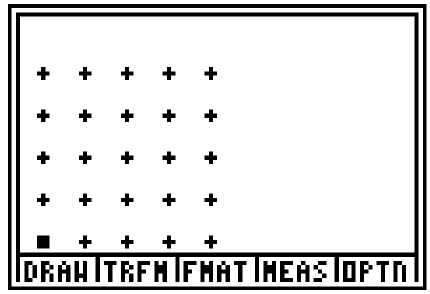




Student Activity

Name ___________________________

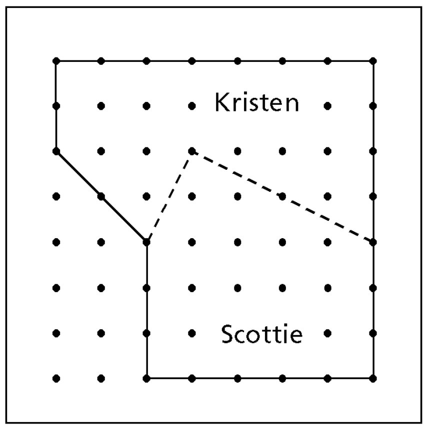
Date ___________________________

Activity 3.4: Out to Pasture

The diagram shows a part of Kristen's and Scottie's pasture property. They share a common fence that needs to be replaced.

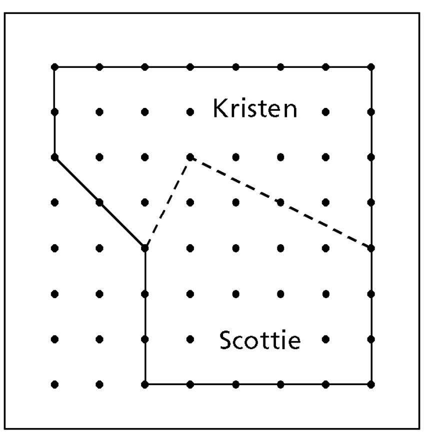
1. What is the area of Kristen's pasture? ___________________

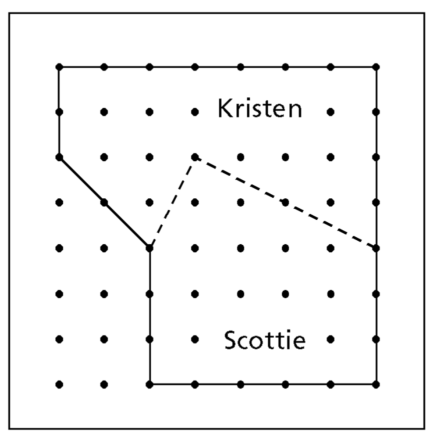
2. What is the area of Scottie's pasture? ___________________



They decide to replace the fence with a different, straight-line fence that will not change the property area of either person. One boundary fence post must stay in the same place.

3. Draw two possible solutions.
   Hint: Use the idea of triangles between parallel lines.





4. Which new fence is shorter? Is it shorter than the original fence?

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____

③ The area is ½ of 1 square unit	③ The shape has 1 obtuse angle
③ The skinny shape has 3 boundary points	③ The shape's height is less than 1 unit
❸ There are no interior points	❸ The shape is a triangle

Teacher Notes

Activity 3

Move Along Little Vertex

Objective

- To use Geoboard to find the areas of triangles having the same base and the same height

NCTM Standards

- Select and apply techniques and tools to accurately find…area…to appropriate levels of precision
- Solve problems that arise in mathematics and in other contexts

Investigation

Moving a triangle's vertex between parallel lines is sometimes called “sheering.” Students should become familiar with the idea that moving a vertex of a triangle along a line that is parallel to the triangle’s base never changes the triangle’s height nor its area. By shifting a triangle’s vertex between parallel lines, we can change any triangle into a right triangle with the same area. Similarly, by shifting any triangle’s base between parallel lines, we can change any triangle into a right triangle with the same area.

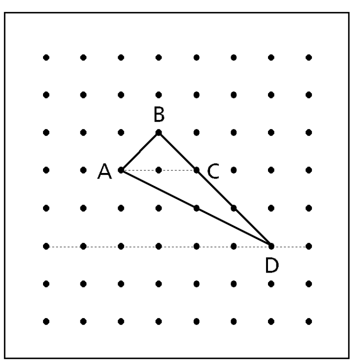
Answers to Student Activity pages

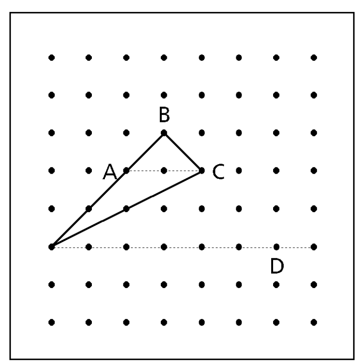
Activity 3.1: Move Along Little Vertex

Triangle	Triangle Base	Length of Base	Height of Triangle	Area of Triangle
XZA	XZ	3	4	6 square units
XZB	XZ	3	4	6 square units
XZC	XZ	3	4	6 square units
XZD	XZ	3	4	6 square units
XZE	XZ	3	4	6 square units

1. The height of the triangle does not change; it is always 4. The vertex always stays on one of two parallel lines that are equidistant everywhere.
2. The area of the triangle does not change; it is always 6. The area depends on the base and the height, which stay the same.
3. Triangle XZB or triangle XZE because they are both right triangles having the same area.
4. The length of the base and the height.
5. Any triangle's area will always be half of the product of its base and height. For right triangles, the area is always half of the surrounding rectangle, which is also half of the base times the height.
6. Right-triangular pen XZB or right-triangular pen XZE.

Activity 3.2: The Quadrilateral Tile Company

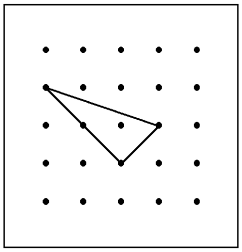




Answers may vary. One possibility is given.

Page 27

1.



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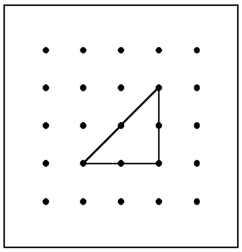

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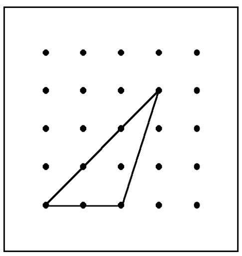

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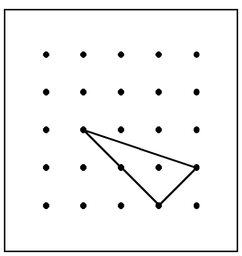
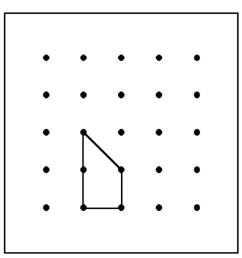


Activity 3.3: The Pentagonal Tile Company

1.



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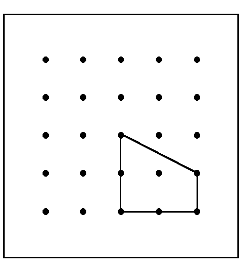

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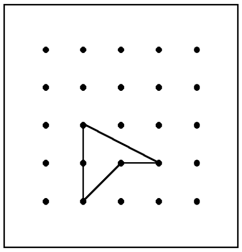

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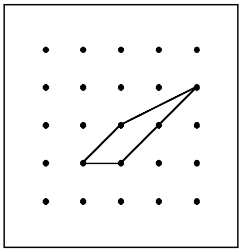
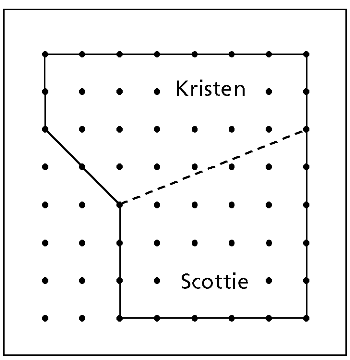
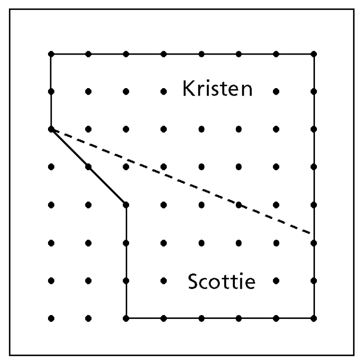


Activity 3.4: Out to Pasture

1. 21 square units
2. 20 square units
3.





4. The one on the left is the shortest and it is shorter than the original fence.

Group Problem Solving: The area of triangles between parallel lines

The Group Problem Solving cards are challenge problems that can be used alone or with the individual sections of this book. The problems are designed to be used in groups of four (five or six in a group are possibilities using the additional cards) with each person having one of the first four clues. Students can read the information on their cards to others in the group but all should keep their own cards and not let one person take all the cards and do the work.

The numbers at the top of the cards indicate the lesson with which the card set is associated. The fifth and sixth clues (the optional clues) have the lesson number shown in a black circle.

The group problems can be solved using the first four clues. The fifth and sixth clues can be used as checks for the group's solution or they can be used as additional clues if a group gets stuck. Some problems have more than one solution. Any shape that fits all the clues should be accepted as correct.

With a little experience, students should be able to design their own group problems. They could then switch problems with other groups for additional problem solving practice.

One solution for this problem solving exercise: