What's the Difference?

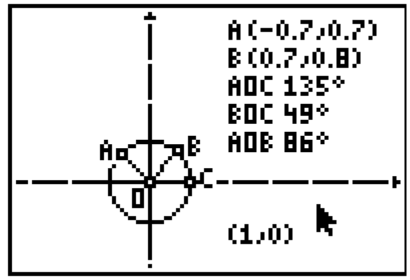
Name ______________________

Class ______________________

Problem 1 – Exploring the Angle Difference Formula for Cosine

Open the *Cabri Jr.* file called **UNITCIRC**. Observe a unit circle with points *A*, *B*, and *C* on the circle. Point *O* is the origin and center of the circle. The central angle ∠*AOB* represents the difference between ∠*AOC* and ∠*BOC*.

Move points *A* and *B* to see the changes to the on-screen measurements. To select an object, press the ALPHA key.



1. The *x*-coordinate of every ordered pair of a point on the unit circle represents the ________________________ of the corresponding angle.

2. The *y*-coordinate of every ordered pair of a point on the unit circle represents the ________________________ of the corresponding angle.

Use the file **UNITCIRC** to answer the following questions.

3. What is the sine of ∠*AOC* when its measure is about 100°?

4. What is the cosine of ∠*AOC* when its measure is about 100°?

5. What is the sine of ∠*BOC* when its measure is about 20°?

6. What is the cosine of ∠*BOC* when its measure is about 20°?

7. What is the sine of ∠*AOC* – ∠*BOC* when *m*∠*AOC* = 100° and *m*∠*BOC* = 20°? Use the Cabri Jr. file to obtain your solution.

8. What is the cosine of ∠*AOC* – ∠*BOC* when *m*∠*AOC* = 100° and *m*∠*BOC* = 20°? Use the Cabri Jr. file to obtain your solution.

9. Do you think the relationship between the values of sine and cosine for $\angle AOC - \angle BOC$ is quickly and easily obtained from the two individual angles as shown on the opening diagram for this activity? Explain you answer.

Problem 2 – Applying the Angle Difference Formula

The opening diagram for this activity is commonly used in the derivation of the angle difference formula for cosine.

$$\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

This formula is useful in finding exact values for the cosine of angles other than those you may already know from the unit circle.

For each exercise below, use your graphing calculator first with and without the formula. Then use the **UNITCIRC** circle graph.

10. Find the value of cos 15° by finding cos(60° – 45°).

11. Find the value of cos 75° by finding cos(120° – 45°).

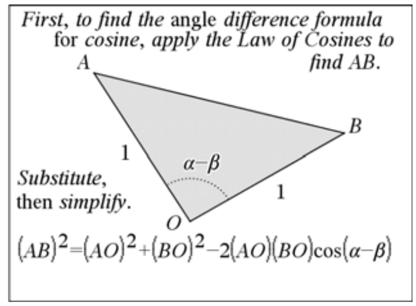
12. Find the value of cos 105° by finding cos(? – ?). You choose the angles! Choose values that you recall from the unit circle.

Extension – Derivation of the Angle Difference Formula for Cosine

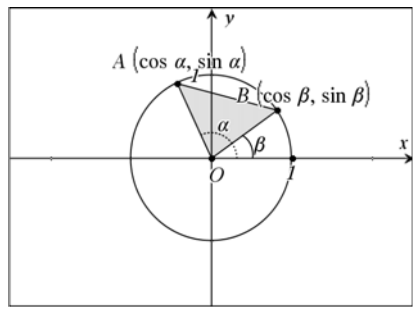
The *Cabri Jr.* file that has been used for this activity will be used in the derivation of the angle difference formula for cosine. As you look at the sketches below, find the angle represented by $\alpha - \beta$. Points *A* and *B* may be moved to change the measures of angles in the diagram.

The angle difference formula for cosine will be derived using the diagrams below.

13. Apply the Law of Cosines to the figure to the right to find an equation representing AB^2.



14. Apply the distance formula to the figure to the right to find an equation representing AB^2.



15. Combine the two equations obtained in Exercises 13 and 14 by setting them equal to each other. Solve for $\cos(\alpha - \beta)$. Test your resulting equation by entering values of your choice in **UNITCIRC**. Does your result agree with the directly calculated angle difference value? If not, check for algebraic and calculator entry errors.

Extension – Derivation of the Angle Sum Formula for Cosine

16. Now substitute $-\beta$ in place of β into the angle difference formula for cosine and simplify the resulting equation. Test your resulting equation by entering values of your choice on page 3.3. Does your result agree with the provided angle sum value? If not, check for algebraic and calculator entry errors.