# NUMB3RS Activity: How Much Can They Win? Episode: "Double Down" 

Topic: Time Series Analysis
Grade Level: 8-10
Objective: Use linear functions to model a time series and predict future values
Time: 20-30 minutes
Materials: graphing calculator

## Introduction

In "Double Down," three students who have taught themselves how to count cards in Blackjack find that they are tied up in a money-laundering scheme, and are fearful of being murdered. When their account records are located, Amita suggests that she and Charlie use a multivariate time series analysis to track the play and flow of money. This activity will investigate a simplified version of a time series analysis.
Suppose that Charlie will be able to look at the "chip buys" and "winnings" of the three students, and then track how they illegally won money from the card club. If Charlie can graph the winnings over time, he may be able to predict future values or future "winnings."

## Discuss with Students

A time series is a mathematical model consisting of a sequence of data points $(x, y)$ where the $x$-values are equally-spaced time intervals. For example, you might record your body temperature every 4 hours to monitor the effectiveness of the cold medicine you took. A time series will usually exhibit a trend over time and often demonstrates cycles within the time period. A graph of the Dow Jones Industrial Average shown over time is a prime example of a time series. The stock prices show a long-term trend of increasing over time, even though there are patterns of highs and lows within the graph. Time series analysis attempts to understand both the patterns within the graph and why they exist, and the overall trend of the graph if it were viewed as a smooth curve. The main goal of time series analysis is to predict future values.

## Student page answers:

1. Assuming that the first win is at hand 4, the profits would be $-50,-100,-150,350,300,250$, 200, 700, 650, 600, 550, and 1,050.

2. There are three $\$ 50$ losses, followed by a $\$ 500$ gain. 3. The winnings steadily increase at a rate of $\$ 350$ every 4 hands. 4. $y=87.5 x, x=\{0,4,8,12, \ldots\}$. 5. $\$ 1,7506.40$ hands 7. $\$ 2,350$, $\$ 5,250$; First, divide the time in minutes by the time for each hand of blackjack. For one hour, they can play 30 hands. Because 30 is not a multiple of 4, use the formula to find the profit for the next greatest number of hands that is a multiple of 4: 28 hands. Then subtract $\$ 50$ for each of the next 2 hands. For two hours, they can play 60 hands. Because 60 is a multiple of 4 , use the formula to find the profit for 60 hands. 8. There is an erratic pattern of increases and decreases because of randomly winning and losing games. 9. It levels out over time to a value of 1.44\%. 10. $\$ 14,400$

Name: $\qquad$ Date: $\qquad$

## NUMB3RS Activity: How Much Can They Win?

In "Double Down," three students have taught themselves how to count cards so that they can win money at the blackjack table in a card club. However, they are soon tied up in a money-laundering scheme and are fearful for their lives. Charlie and Amita use time series analysis to track the play and flow of money so that they can predict the students' and the killers' next steps. Suppose the students have found a way to hack into the card shuffle track used at the blackjack table, and know that for a fixed period of time they will win every fourth hand. On the hands that they lose, they lose $\$ 50$, and on the hands that they win, they win $\$ 500$.

1. Complete the table, and sketch a graph showing their profit over 12 hands.

| Hand <br> Number | Profit <br> (in dollars) |
| :---: | :---: |
| 0 | 0 |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |
| 11 |  |
| 12 |  |


2. Describe the pattern that occurs during every four-hand cycle.
3. Describe the long-term trend of winnings. $\qquad$
4. Charlie wants to predict the future values for their winnings.
a. Find an equation of a line that contains all of the high points within each 4-hand cycle. Let $y$ represent the winnings, and $x$ represent the number of hands played. $\qquad$
b. State the domain of the function.
5. Predict the students' profit after playing 20 hands.
6. How many hands will the students have to play to win $\$ 3,500$ ? $\qquad$
7. If one hand of blackjack takes 2 minutes to play, what is their profit if they play for exactly one hour? Two hours? Describe how you calculated the profit.

In a fair game of blackjack, where neither the card club nor the player cheats, the blackjack payout is $98.56 \%$. This means that for every $\$ 100$ bet, $\$ 98.56$ goes to paying off winnings, and the remaining $\$ 1.44$ is the card club's profit. In other words, the card club can expect to net a profit of $1.44 \%$ over time. Suppose the graph below is a time series graph that shows the percent profit over a 24-hour time period for all of the blackjack games taking place in the club. The $x$-axis represents time in hours with a scale of 4 hours, and the $y$-axis represents the percent profit for the card club, with a scale of 1.

8. Based on the patterns you see in the graph, make a conjecture about what happens to the profits during the first 8 hours of a typical 24-hour period.
$\qquad$
9. Describe the card club's profits over the 24 -hour period.
10. If the card club averages blackjack bets totaling $\$ 1$ million each day, how much daily profit would they expect to make? $\qquad$

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

## Extensions

## For the Student

A graph of the Dow Jones Industrial Average shown over time is a prime example of a time series. The stock prices show a long-term trend of increasing over time, even though there are patterns of highs and lows within the graph.

- Locate a time series graph for the US stock market over the last 12 months. Use the graph to find a mathematical model that you can use to predict the value of the stock market one year from now.
- Locate a time series graph for the US stock market from 2002 until now. Use the graph to find a mathematical model that you can use to predict the value of the stock market one year from now.
- Locate a time series graph for the US stock market for the last 50 years. Use the graph to find a mathematical model that you can use to predict the value of the stock market one year from now.
- Explain the differences in predictions and the circumstances where each graph and prediction might be useful.


## Additional Resources

Visit the Web site below for another time series activity. In this activity, students examine the United States Census Bureau Web site to investigate projections of the total population of states from 1995-2025. Using the provided data, students analyze statistics from five states of their choice, develop specific research questions using the data, and create three graphs to compare and contrast the information.
http://illuminations.nctm.org/index_d.aspx?id=239
This website contains an article about time series analysis. It provides several different examples of time series graphs, their meanings, and how they are used. It was produced for the Australian Bureau of Statistics, an agency of the Australian government. http://www.abs.gov.aulAusstats/abs@.nsf/Lookup/CFA19371D1BFAB40CA256F2A 000FEB10

The Math Forum website offers several activities for both middle school and high school students to learn more about how the stock market works. One of the activities is called the Stock Market Game. The Stock Market Game (SMG2000) is an electronic simulation of Wall Street trading, designed to help students and adults understand the stock market, the costs and benefits involved in decision-making, the sources and uses of capital, and other related economic concepts. Since its introduction in 1977, the SMG program has become one of the most widely used classroom tools for teaching students about the American free-enterprise system. In connection with an appropriate course, students in grades 4 through 12 form teams to compete against each other. Competitions consist of teams investing a hypothetical \$100,000 in stock. http://mathforum.org/library/topics/stock_mkt/

