

TEACHER NOTES

Lesson Overview

In many instances, multiple people rate products or people, and these ratings then have to be compiled into a single rating to provide a rank order for the entities being rated. This activity uses the ratings made by five judges in a poster competition with the goal of deciding the top three posters based on the ratings made by the judges.

Learning Goals

Students will be able to:

- 1. Model a contextual situation mathematically and use the model to answer a question.
- Create equations in two or more variables to represent relationships between quantities.
- 3. Summarize and compare data sets using measures of center and variability.

About the Lesson and Possible Course Connections:

The activity can be used with middle and introductory algebra students, and lends itself to a group project. The activity could be a precursor to a more extensive activity on investigating voting methods for students in geometry or second year algebra. The analysis can be connected to statistical summaries for center and variability and possibly to *z*-scores, as a way to normalize the ratings of the different judges.

CCSS Standards

Algebra Standards

- HSA.CED.A.1
- HSA.CED.A.2
- HSA.CED.A.3

Interpreting Data Standards

- 6.SP.B.4
- 6.SP.B.5
- HSS.ID.A.1

Mathematical Practice Standards

• SMP.4

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Lesson Materials

• Compatible TI Technologies:

TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Judging a Poster Contest_Student.pdf
- Judging a Poster Contest_Student.doc
- Poster Contest Data.tns
- Judging a Poster Contest_Teacher.pdf
- Judging a Poster Contest_Teacher.doc

Background

An international association sponsors a poster competition for secondary students from countries across the world. Countries interested in participating organize the competition for teams of students in their country, and the winning poster is entered into the international competition as that country's submission. The posters are then rated by five international judges on six criteria (clarity of message, data collection, analysis, graphs and tables, presentation, creativity/importance) with the higher score the better. The maximum number of points any team can get is 31. The top three scores receive a significant monetary prize.

Facilitating the Lesson

Teams of upper secondary students from 29 different countries entered the poster contest in a given year. The table contains the total number of points given by the judges for each poster. How would you combine the ratings to determine the top three posters?

Upper Secondary Competition

Country	Judge Alpha	Judge Beta	Judge Kappa	Judge Delta	Judge Zeta
A	25	30	22	24	23
В	28	20	22	19	30
С	29	25	20	12	29
D	25	25	24	16	24
E	27	19	25	15	28
F	25	16	27	20	25
G	23	18	26	12	31
Н	28	15	26	9	29
I	22	19	23	19	23
J	24	20	25	9	26
K	23	24	24	8	25
L	21	25	23	10	21
Μ	27	22	22	10	18
Ν	25	17	20	6	29
0	26	16	24	9	20
Р	18	15	22	9	29
Q	21	22	21	9	19
R	25	14	20	10	19
S	19	19	21	9	20
Т	19	23	22	0	24
U	20	17	21	11	18
V	17	13	25	15	15
W	22	17	21	10	15
Х	19	17	20	6	22
Υ	21	17	19	10	14
Z	10	17	17	20	15
AA	17	12	24	10	16
BB	20	12	19	7	12
CC	6	13	16	2	16

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1) Open-Ended Approach:

Have the students think alone for a few minutes and then write down how they would start the task. Have them work in pairs. Students should open the .tns file with the data. If they enter the data by hand, they are likely to make errors.

Have students brainstorm possible strategies, choose one, and use it to answer the question.

2) More-Structured Approach to Finding a Model:

Students should:

- 1. Write down what they observed about the numbers when they were entering them (e.g., Judge Delta had a lot of low ratings, even a zero; the lowest rating Judge Kappa gave was 16; only one country, Z, had a perfect score.)
- 2. Decide whether all of the judges seemed to score the same by finding some measure of center and variability for each of the five judges (median/box plot or mean/mean absolute deviation or standard deviation).
- 3. Each student in the group should write down an approach for compiling the numbers, then in a round robin, share their approaches. As a group discuss the advantages and disadvantages of the proposed strategies (e.g., adding up all of the ratings seems unfair as some judges gave more points than others; throwing out the highest score throws out a lot of the scores given by one judge).
- 4. Decide on a strategy and use it to compile the ratings to produce a ranking that will identify the three countries with the top posters.

What to Expect: Example Student Approaches

Possible strategies:

- Throw out the highest or lowest scores or both.
- Use the mean and either the mean absolute deviation or standard deviation to eliminate any scores that were outliers for a particular judge, then take the average of the remaining scores per country.
- Rank order the ratings for each judge from 1 to 29 and add the rank orders.
- Take the maximum score by a judge as 100% and then find what percent the judge's other scores were of that score. For example, if a top score for any country by that judge is 29 (100%), a score of 25 would be calculated as 25/29 or 86%. This percentage would be computed for every score for that judge. Repeat the process for each judge and then compile the percentages for each country.
- More advanced students might transform each judge's scores into *z*-scores so they are all scaled relative to the same measure, and then combine the *z*-scores for a total.
- Groups may come up with other strategies as well noting that in this case, the only thing that matters is the top group of scores, considering just the countries in the upper quartiles for each judge's scores.

Teacher Note: The situation can be modeled by a logistic curve, but it is not necessary for students to formalize the relationship as logistic.

✤ Validating the Models

Students should validate their models by asking whether the models make sense in different scenarios related to the context as described below.

When students have come up with their strategy and ranking, give them the following task.

Use your method on the points given by the judges for the teams of lower secondary students for each country. Note that the countries were slightly different with six countries not entering a team in this division and one new country that only entered a team in this division. Does your method for ranking still make sense or would you like to adjust it? Explain how and why.

Lower Secondary Competition

Country	Judge Alpha	Judge Beta	Judge Kappa	Judge Delta	Judge Zeta
Δ	27	28	25	22	25
	27	20	20	22	18
<u>ј</u>	21	24	10	20	25
1 S	20	20	19	20	2J 10
5	20	20	23	40	10
G	20	20	20	13	29
DD	27	18	23	20	19
К	22	18	25	12	27
0	25	13	27	11	27
В	24	22	23	14	14
E	22	22	26	11	15
С	20	20	21	17	16
L	19	20	21	13	21
AA	26	18	23	5	22
Р	23	22	18	16	13
V	11	14	22	21	18
U	24	17	18	7	18
Z	19	23	16	10	15
R	22	15	19	6	19
Y	12	15	21	12	18
I	20	14	20	5	14
Ν	21	11	13	1	13
CC	9	10	20	3	14
Н	10	9	18	5	13
Х	11	12	17	3	10



Extension

- 1. Events in the International Olympics Gymnastics competitions are scored by individual judges. Research how the scores are compiled to obtain an overall score and how the current method for scoring came about.
- 2. A group of 21 voters are deciding which of four candidates, A, B, C, and D, should be elected. Each of the 21 voters ranked the 4 candidates from best to worst (not allowing ties). The results are in the table, where the first row indicates three voters ranked A as first, B second, C third, and D fourth. Given these opinions from the voters, which candidate should win the election? Explain your reasoning. (Check the source below to read an analysis of the problem.)

Number Voters	Ranking
3	ABCD
5	ACBD
7	BDCA
6	CBDA

Source: Pacuit, Eric, "Voting Methods", The Stanford Encyclopedia of Philosophy (Fall 2019 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2019/entries/votingmethods/>.

3. In most elections for public office in the United States, the winner is the candidate with the most votes, a method called Plurality. Look up voting methods or theories and write a short report describing other methods of voting (such as Borda Count, Plurality with elimination, Pairwise Comparisons).