LESSON

0.5

PLANNING

LESSON OUTLINE

One day:

35 min	Investigation with sharing
5 min	Closing
10 min	Exercises

MATERIALS

- dice (one per pair)
- centimeter rulers or rulers cut from the transparency Centimeter Rulers (one per pair)
- A Chaotic Pattern? (W, one per pair)
- transparencies and erasable transparency markers (one per pair)
- Calculator Notes 0E, 0F, 0G

TEACHING

In this lesson students discover that a fractal is an attractor of a random recursive process.

Guiding the Investigation

Every class will benefit from following the steps given in the text. Organize students into pairs. Give each pair of students the worksheet, a transparency, and a transparency marker.

Steps 1 to 4 You may want to demonstrate the first several steps in the Chaos game on the board or overhead projector, explaining the random element of rolling a fair die. Model careful measurement and care in remembering the location of the last dot marked. The first three or four dots might not fit the pattern, so after you have marked four points, erase the first three. Be sure students follow the directions to switch tasks after the first 20 points are marked.

LESSON 0.5

Out of Chaos

Nothing in nature is random. . . .
A thing appears random only
through the incompleteness
of our knowledge.
BARUCH SPINOZA

If you looked at the results of 100 rolls of a die, would you expect to find a pattern in the numbers? You might expect each number to appear about one-sixth of the time. But you probably wouldn't expect to see a pattern in when, for example, a 5 appears. The 5 appears **randomly,** without order. You could not create a method to predict exactly when or how often a 5 appears. As you explore seemingly random patterns, you'll review some measurement and fraction ideas.

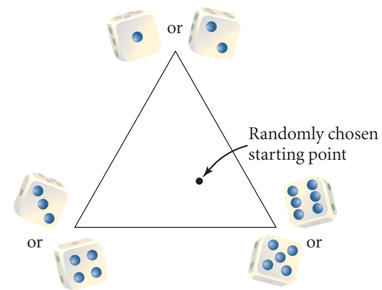
Investigation
A Chaotic Pattern?

You will need

- a die
- a centimeter ruler
- a blank transparency and marker
- the worksheet A Chaotic Pattern?

What happens if you use a random process recursively to determine where you draw a point? Would you expect to see a pattern?

Work with a partner. One partner rolls the die. The other measures distance and marks points.



For best results, measure as accurately as you can.

Step 1 Mark any point inside the triangle as your starting point.

Step 2 Roll the die.

Step 3 In centimeters, measure the distance from your starting point to the corner, or *vertex*, labeled with the number on the die. Take half of the distance, and place a small dot at this midpoint. This is your new starting point.

Step 4 Repeat Steps 2 and 3 until you've rolled the die 20 times. Then switch roles with your partner and repeat the process another 20 times.

Step 5 How is this process recursive? Answers should indicate that the result from one step becomes the input for the next step.

Step 6 Describe the arrangement of dots on your paper. Answers should mention that some areas show lots of dots while others remain empty. Some might mention Sierpiński's triangle.

Step 7 What would have happened if you had numbered the vertices of the triangle 1 and 3, 2 and 5, and 4 and 6? Answers should indicate that the result would be the same.

Step 2 **[Language]** *Die* is singular, *dice* is plural.

Step 3 **[Language]** Use the term *vertex* for a corner of the triangle and *vertices* for the plural.

These four steps take about 10 minutes to complete. Student results will look pretty chaotic at this point, although students may notice a lack of dots in the centers of their triangles. To keep students motivated, you might want to remind them that larger collections of data often show more patterns than smaller sets. Hint that they will eventually see an unexpected pattern.

Step 5 The procedure is recursive: At each stage students are building on the point that results from the previous stage.

Step 8 Place a transparency over your worksheet. Use a transparency marker and mark the vertices of the triangle. Carefully trace your dots onto the transparency.

Step 9 Since the position of a dot is used as input for determining the next dot, the process is recursive.

Step 9 When you finish, place your transparency on an overhead projector. Align the vertices of your triangle with the vertices of your classmates' triangles. This allows you to see the results of many rolls of a die. Describe what happens when you combine everyone's points. How is this like the result in other recursion processes? Is the result as random as you expected? Explain.

A *random* process can produce ordered-looking results while an orderly process can produce random-looking results. Mathematicians use the term *chaotic* to describe systematic, non-random processes that produce results that look random. Chaos helps scientists understand the turbulent flow of water, the mixing of chemicals, and the spread of an oil spill. They often use computers to do these calculations. Your calculator can repeat steps quickly, so you can use the calculator to plot thousands of points.

Step 10 Enter the Chaos program into your calculator. [▶ See **Calculator Note 0E** for the program. To learn how to link calculators, see **Calculator Note 0F.** To learn about how to enter a program, see **Calculator Note 0G.**◀]

The program randomly "chooses" one vertex of the triangle as a starting point. It "rolls" an imaginary die and plots a new point halfway to the vertex it chose. The program rolls the die 999 more times. It does this a lot faster than you can.

Step 11 Run the program. Select an equilateral (equal-sided) triangle as your shape. When it "asks" for the fraction of the distance to move, enter $\frac{1}{2}$ or 0.5. It will take a while to plot all 1000 points, so be patient.

Step 12 What do you see on your calculator screen, and how does it compare to your class's combined transparency image? Answers should mention Sierpiński's triangle and the similarity to the combined points for the class transparencies.

Most people are surprised that after plotting many points, a familiar figure appears. When an orderly result appears out of a random process like this one, the figure is a *strange attractor*. No matter where you start, the points "fall" toward this shape. Many fractal designs, like the Sierpiński triangles on your calculator screen, are also strange attractors. Accurate measurements are essential to seeing a strange attractor form. In the next example, practice your measurement skills with a centimeter ruler.

Science CONNECTION

The growth and movement of an oil spill may appear random, but scientists can use chaos theory to predict its boundaries. This can aid restraint and cleanup. Learn more about the application of chaos theory with the Internet links at www.keymath.com/DA .

Step 9 As students add more layers of transparencies, they will begin to see a familiar pattern appear. It can be very exciting to see the Sierpiński triangle appearing out of the chaotic mass of dots. The Sierpiński triangle is an attractor for the Chaos game.

If a few dots don't fit the pattern, they were probably some of the first few dots a group marked.

Step 10 Students need Calculator Notes 0E, 0F, and 0G.

Step 11 You might challenge students to try various shapes and fractions and report their results.

SHARING IDEAS

Most reporting in this lesson takes place during the investigation. You need not push for deep understanding of the two terms defined here informally: *random* and *predict*. They will be used again later.

[Ask] "Did the game result in a random pattern?" [No, the process was random, but the overall result is predictable.]

You might mention that, technically, the Chaos game is misnamed; mathematical chaos is the production of seemingly random patterns by methods that are not random.

The random generation of the Sierpiński triangle gives experience with careful measurement and more practice at fraction multiplication and recursion.

Assessing Progress

To complete the investigation, students need to know what is meant by the **vertex** of a triangle and the **midpoint** of a segment. And they must be able to multiply fractions or decimals to find the midpoint of the segment between the vertex of the triangle and a point. They should understand that the Chaos game uses **recursion** and the resulting Sierpiński triangle is an **attractor.**

LESSON OBJECTIVES

- Practice multiplication with fractions and decimals
- Deepen understanding of recursion and attractors
- Practice careful measuring skills
- Review the meaning of a vertex of a triangle
- Learn to link calculators to transfer a program
- Learn to run a calculator program

NCTM STANDARDS

	CONTENT		PROCESS
	Number		Problem Solving
	Algebra	•	Reasoning
•	Geometry		Communication
•	Measurement	•	Connections
	Data/Probability	•	Representation