NUMB3RS Activity: Thumbs Up!
Episode: “Mind Games”

Topic: Using simulations to understand variability **Grade Level:** 9 - 12
Objective: Learn which outcomes are likely to occur by chance based on a simulation, and what it means to have rare or surprising events.
Materials: TI-83/84 Plus calculator or a large random number table
Time: 20 - 30 minutes

Introduction

In “Mind Games,” the FBI hires a psychic to help them solve a series of murders. To test how good he is, they show him the backs of a set of 25 playing cards and ask him to predict whether each is red or black. He gets every single one wrong.

Because the probability of incorrectly guessing the color of each card is 50%, one would expect that guessing about half of them incorrectly is reasonable. If he is a psychic, perhaps he should have guessed more correctly. However, you would probably still be surprised if he guesses all of them correctly or all of them incorrectly.

The idea of this activity to show students that there is a range of values which are acceptable and within a certain level of expectation or chance and which may be more of a rare occurrence.

In this activity, students conduct an experiment and then use a simulation to construct an interval based on chance that contains 90% of the outcomes for the experiment. The simulation will introduce them to the concept of variability and the idea of a rare event.

Discuss with Students

Students are probably used to hearing statements in the news such as “The average cost of gasoline is between $2.75 and $3.50 a gallon” or “the average age of a college student is between 18 and 23 years old.” With many everyday occurrences, there is a range of values that seem reasonable which also helps us identify values that do not seem reasonable. In our examples above, it is reasonable to find the average gas cost in the sample to be $2.95, or a college student to be 19 years old. You may question its validity if an average gas price from the same sample is $5.00 a gallon for example. In this activity, you will be generating data based on a given parameter and finding the range of comfort. You will also be identifying which numbers are considered rare or outside of what is an expected range of values.

Motivational Activity

Ask students to fold their hands so that their fingers are interlaced comfortably. Ask them to look at where their thumbs are. Is the thumb on top the right or the left thumb?

Have students guess what percent of the class folded their hands with their right thumbs up. They may say 50% because it is equally likely that someone has thumbs up as down. Students may also say that it is not 50% because there are more right-handed people, so there are more thumbs up with the right hand. Others may say that it is whatever feels natural, and that some right-handed people may prefer left thumb up. The discussion will conclude that there is really no way to tell, but there is some range of probability you can say with a level of confidence.

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For example, students will understand that the occurrence that 100% of the students in the class having right thumbs up is possible but rare, whereas a range of 30% to 50% could more be reasonable.

In this activity, students are given that 30% of people fold their hands with the right thumb up. Using this parameter, students will determine the upper and lower cut offs in the distribution that contains at least 90% of the data. Data that falls outside of the 90% would be considered a rare occurrence.

Because this activity uses a simulation, actual answers will probably vary from any "answer key," so a sample distribution is included, along with answers to the activity questions as they pertain to the sample. To shorten time needed for this activity, the sample distribution can be used.

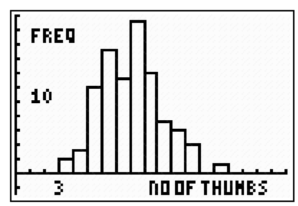
The activity is designed around first actually doing the experiment with 25 students. If your class is larger than 25, count the first 25 students; if smaller, borrow a few volunteers. For the simulation, a total of 100 samples, each of size 25, is needed. Divide the class to produce a total of 100 samples, and then pool the samples into a single frequency table.

Student Page Answers:

1. $(0.5)^{25} \approx 2.98 \times 10^{-8}$

Below is a sample distribution for 100 trials with 25 people in each trial. (For example, the 21 below the 8 means that in 21 of the 100 trials, exactly 8 people had their right thumb on top.)

# of Right Thumbs	0	1	2	3	4	5	6	7	8	9
Frequency	0	0	0	2	3	12	17	13	21	14
# of Right Thumbs	10	11	12	13	14	15	16	17	18	19
Frequency	7	6	4	0	1	0	0	0	0	0



All answers from this point forward will vary. The answers shown are for the above distribution.

2. *Answers vary. Sample answer: from 5 to 12 incorrect guesses might be reasonable; other answers might be considered suspicious.* **3a.** *Answers will vary.* **3b.** *Not necessarily. If* ***many*** *samples were considered, we would expect 30% of 25, or 7.5 to have the right thumb on top. Though not close to 7.5, 15 is possible in a single sample.* **4a.** *Because there are ten digits, 0, 1, and 2 represent 30% of the digits. You could choose any three of the ten digits to count as "right thumb up."* **4b.** *A sample distribution and histogram are shown above.* **5.** *Answers will vary but can be based on how many times the class percentage occurred in the 100 samples.* **6.** *For the sample distribution, the interval from 5 to 11, inclusive contains 88% of the samples.* **7.** *The interval contains 88% of the sample.* **8.** *Answers will vary.* **9.** *Narrower. The larger the sample, the more the data will "pile up" closer to the expected value. This in turn will make the intervals for the upper and lower 5% wider.* **10.** *Wider. By having to include a higher percentage of the data, the range will have to include more values.* **11.** *To construct a simulation, because the probability of guessing the color is 50%, set the simulator to generate sets of 5 zeros and ones. Let one of the digits (say 1) represent a correct guess. Generating five sets of numbers at a time and counting the 1s is equivalent to guessing the correct color of a group of cards from a set of 25 cards.*

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Name: ________________________________ Date: ________________

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A natural question to ask when considering outcomes to an experiment is, “Are you surprised?” Some outcomes are simply due to chance. Other outcomes may seem out of the ordinary, and you may want to examine what is happening more carefully. In “Mind Games,” the FBI hires a psychic to help them solve a series of murders. To test how good he is, they show him a set of 25 playing cards one at a time. He is shown only the backs of the cards and asked to predict whether each is red or black. He gets every single one wrong.

Would you expect this? What is the probability of guessing them all wrong? An even better question is “what is a reasonable range of likely incorrect guesses before the results begin to look suspicious?” In other words, how much variability would you expect from just chance if he simply guessed at each card?

1. If the probability of incorrectly guessing the color of a single card is 0.5, what is the probability of guessing all 25 incorrectly?

2. Describe a range of incorrect guesses that you would think would be reasonable for the psychic to obtain. How many incorrect guesses could he make before you would be suspicious?

Typically, a range of values like the number of incorrect guesses can be used to describe a certain amount of reasonableness. So, if someone guessed out of 25 times, 12 correct and 13 incorrect, you would say that is within a reasonable range, and something that you may expect. You, for example may not expect someone to guess them all 25 right or all wrong. In this activity, you will be collecting data and finding the range of comfort for the data and identifying what would be outside the range and considered to be a rare occurrence.

3. Consider the following experiment: fold your hands so that your fingers are interlaced comfortably, and notice which thumb is on top.

a. Count the results for your class. Record the percentage of “right thumbs up.”

b. Suppose that you knew that approximately 30% of all people folded their hands with the right thumb on top. If you sampled a group of 25 people and found that 15 of them (60%) had their right thumbs on top, would this mean that you had made an error in counting or that the original 30% is not right? Explain.

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Follow the directions below to simulate taking a sample of 25 people by finding five groups of five random numbers. The application only shows five values at a time, so to generate a random number for each of the 25 people, run the simulation 5 times.

On your calculator, press [APPS] and scroll down to `Prob Sim` (Probability Simulation). Press any key to start the application; then select `6.Random Numbers`. Press **Set** (for settings) and configure the simulator to use 5 numbers in the range 0 – 9 and choose **Yes** for Repeat (see Fig. 1 below). Press **OK**.

Fig. 1: Random Number settings:

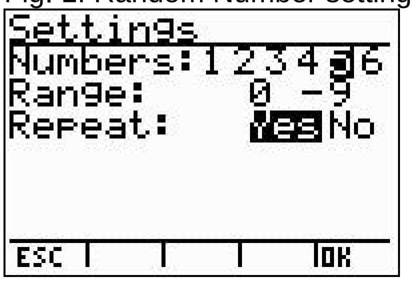
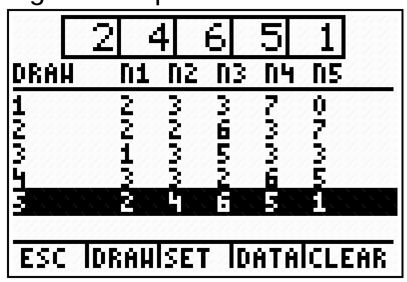


Fig. 2: Sample data set:



Let's assume that 30% of people fold their hands with "right thumbs up." Let the digits 0, 1, and 2 represent "right thumb up," and all others represent "left thumb up." Press [ENTER] or **DRAW** a total of five times, then count the number of 0s, 1s, and 2s for all five runs displayed on the screen (see Fig. 2 above). Record the number of "right thumbs." In this example, 8 of the 25 random numbers are 0, 1, or 2, meaning that 8 people had "right thumb up." At your teacher's direction, press **CLEAR** and repeat the simulation so your class generates a total of 100 samples.

4. a. Explain why 0, 1, and 2 are selected to represent "right thumbs up" in this simulation. Is there a different set of digits that could represent "right thumbs up?"

b. Tally the class's results in the table below, and create a histogram of the data.

# of Right Thumbs	0	1	2	3	4	5	6	7	8	9	10	11	12
Frequency													
# of Right Thumbs	13	14	15	16	17	18	19	20	21	22	23	24	25
Frequency													

5. Compare your class percentage of thumbs up found in question 3a with the distribution you generated by chance. Does your class percentage seem surprising or not surprising? Explain.

6. Look at your distribution. Find the interval of "right thumbs up" that contains as close to 90% of the samples as possible, excluding the lower 5% (or less) and the upper 5% (or less). [Hint: There are 100 samples, so exclude the bottom 5 samples and the top 5 samples. Make sure that you include the entire frequency for each number of right thumbs.]

7. How many of the values are in the interval determined by your answers for question 6? What percentage of the total outcomes would this be?

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Suppose an outcome that occurs at least 90% of the time by chance is considered not surprising. Also if an outcome occurs less than 10% of the time due to chance, it is surprising. Thus if your outcome falls in the lower or upper 5% of the values in the distribution, the outcome is considered unlikely to happen by chance. Remember that these outcomes *can happen by chance 10% of the time*.

8. Based on your 90% interval for likely outcomes due to chance, is the class percentage for “thumbs up” surprising? Explain.

9. If your 90% interval had been determined by using samples of size 100 or 1,000, would you expect the interval to be wider or narrower? Explain.

10. Suppose you chose to make a rule for likely or unlikely using at least 95% of the outcomes rather than 90%. Would the interval likely to occur by chance be wider or narrower than the interval you found for 90%? Explain.

11. Optional Create a simulation and construct an interval containing at least 90% of the number of correct (or incorrect) guesses that would be likely from guessing the color (red or black) of 25 playing cards. If someone guessed the color of 18 of the cards correctly, would you be surprised? Why or why not?

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

This activity was designed to demonstrate that when randomly sampling a population, there is variability in the samples. Even though the probability was 30%, the sample results varied. None of the sample results were exactly 30% of the sample. However, the results did tend to cluster around the true probability. For example, from the sample data, 83 samples fell between 5 and 10, inclusive. This activity was meant to be a general introduction to inference.

For the Student

- You can also use the **randInt(** command on a TI-83/84 Plus calculator to generate the simulation in the activity. Investigate how you can use lists to create a histogram of the results.
- How would the simulation change if the probability of folding your hands with your right thumb up was 0.4?
- How would the simulation change if you looked at samples of 50 people?
- Suppose you toss a coin 1,000 times. Would you be surprised if the coin landed on heads 550 times? 600 times? 900 times? When would you begin to believe that either the coin or the toss might not be “fair?”

Additional Resources

- To make tables of random digits, see: **http://www.random.org/nform.html**
- The Probability Simulation App can be downloaded for free from: **http://education.ti.com/us/probsim**
- For a more detailed look at computing the probability that the psychic could actually “guess” the color of all 25 cards incorrectly, see the activity **Right or Wrong**, which also accompanies the *NUMB3RS* episode “Mind Games.” This activity can be downloaded for free from the Web site below. **http://www.cbs.com/primetime/numb3rs/ti/activities/Act2_RightOrWrong_MindGames_final.pdf**
- This book offers more information and activities relating to Data Analysis.

  Burrill, Gail, Landy Godbold, Christine Franklin, and Linda Young, *Navigating Through Data Analysis for Grades 9-12.* J. Lott (ed.) Reston, VA: National Council of Teachers of Mathematics, 2004.
- This book offers more information and activities relating to Probability.

  Barrett, Gloria, Rick Billstein, Henry Krandondonk, Roxy Peck, Michael Shaughnessy, *Navigating Through Probability for Grades 9-12.* J. Lott (ed.) Reston, VA: National Council of Teachers of Mathematics, 2004.

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