

String Art

Teacher Notes and Answers

7 8 9 10 11 12



Introduction

In this task, you will learn how a routine topic in coordinate geometry is, with the aid of graphing technology, extended to explore deeper concepts of variable, function and transformation. It is based on a craft art idea popular in the 1960's and 1970's. (Search Youtube for "String Art").

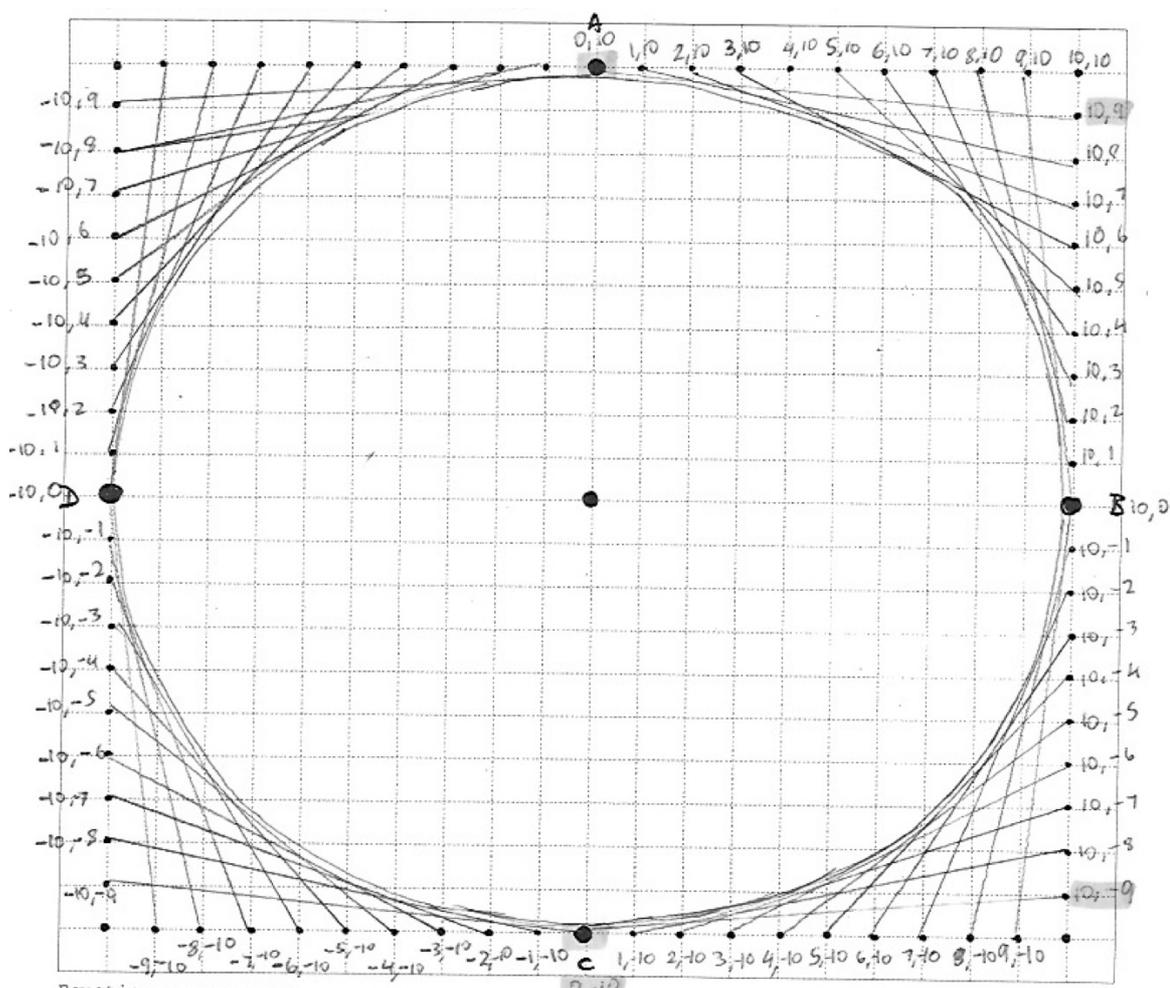
Step 1: Construct the string art design manually

Question 1.

Complete a manual string art design of a circle as per teacher instruction on the grid below.

- Draw a line from Point A to the second point on the right edge of the square.
- Continue this pattern until the middle of the grid appears to be 'circular'.

Student sample of a basic string art design – some teacher guidance may be needed here.



Step 2: Construct rules for each line (1st quadrant)

Now suppose we overlay a Cartesian plane, with the origin at the centre, and the grid spacing at 1 unit. For example, Point A has coordinates (0,10) and Point B has coordinates (10,0).

Question 2.

Mark the coordinates of all the relevant points in the first quadrant.

Constructing rules from two points

Recall that for the line with equation $y = mx + c$ passing through the points (x_1, y_1) and (x_2, y_2) , the gradient (m) and the y -intercept (c) can be found using $m = \frac{y_2 - y_1}{x_2 - x_1}$ and $c = y_2 - mx_2$

For example, the student work sample shows how the equation of the line joining the points (1,10) and (10, 8) can be found.

Handwritten student work showing the calculation of the equation of a line passing through (1, 10) and (10, 8). The work includes the formula $y = mx + c$, the calculation of the gradient $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 10}{10 - 1} = -\frac{2}{9}$, and the calculation of the y -intercept $c = y_2 - mx_2 = 8 - (-\frac{2}{9})(10) = 8 + \frac{20}{9} = \frac{72}{9} + \frac{20}{9} = \frac{92}{9}$. The final equation is $y = -\frac{2}{9}x + \frac{92}{9}$.

Question 3.

Using the method shown right, calculate the equation for each line drawn in this quadrant. Write the 10 equations in the space below.

This can take a while, so consider setting it as a homework task. It is important that students do this work manually before the next part of the investigation

(x_1, y_1)	(x_2, y_2)	Equation
(0,10)	(10,9)	$y = -\frac{1}{10}x + 10$
(1,10)	(10,8)	$y = -\frac{2}{9}x + \frac{92}{9}$
(2,10)	(10,7)	$y = -\frac{3}{8}x + \frac{86}{8}$
(3,10)	(10,6)	$y = -\frac{4}{7}x + \frac{82}{7}$
(4,10)	(10,5)	$y = -\frac{5}{6}x + \frac{80}{6}$
(5,10)	(10,4)	$y = -\frac{6}{5}x + \frac{80}{5}$
(6,10)	(10,3)	$y = -\frac{7}{4}x + \frac{82}{4}$
(7,10)	(10,2)	$y = -\frac{8}{3}x + \frac{86}{3}$
(8,10)	(10,1)	$y = -\frac{9}{2}x + \frac{92}{2}$
(9,10)	(10,0)	$y = -10x + 100$

Step 3: Check your string art design via technology (1st quadrant)

We will now use the TI-Nspire CAS to check the accuracy of calculated lines for the first quadrant of the design.

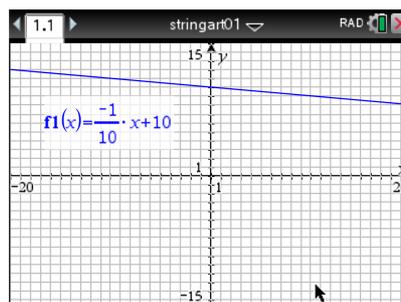
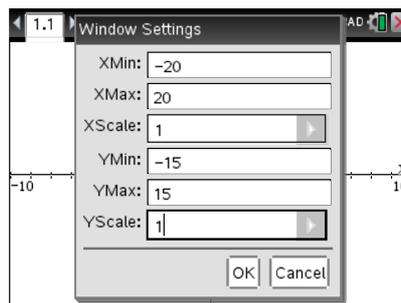
To do this using the TI-Nspire CAS

- Press  > **New Document**, and then select **Add Graphs**.
- Press  > **Window/Zoom** > **Window Settings** to make the graph screen have dimensions $[-20,20]$ by $[-15,15]$, and the **XScale** and **YScale** values as 1 (see correct values shown right).
- Press  > **View** > **Grid** > **Lined Grid** to make a lined grid visible
- Press   and then enter the rule for the first line

Let the first line have equation $y = \frac{-1}{10}x + 10$

The graph screen should appear as at right (Note that the function 'f(x)' is used here instead of 'y')

- To make it easier to view the graphs without the labels cluttering the view, press  > **Settings**, and check the option to 'Automatically hide plot labels'.



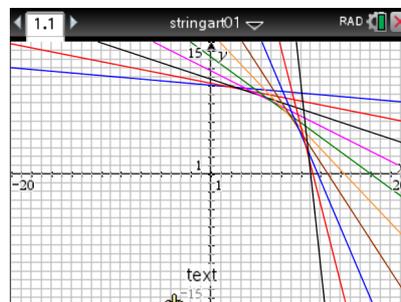
Question 4.

Use the above processes to enter and graph the 10 straight lines that you found in Question 3.

Make sure that make a pattern forms similar to the one shown in your manual design from

Question 1 (except that the lines continue in both directions here!).

The order of graph entry is not critical, but preserving some order makes it easy to see the pattern in the changes for the gradient and y-intercept. The following screens show the expected result.



Step 4: Building a set of lines (teacher-led)

Observe the equations of the lines you have constructed – might there be a way to generate the above set of lines by looking at the set of values of x_1 , x_2 , y_1 , y_2 being used? Here are the pairs of points for each line found previously.

	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6	Line 7	Line 8	Line 9	Line 10
(x_1, y_1)	(0,10)	(1,10)	(2,10)	(3,10)	(4,10)	(5,10)	(6,10)	(7,10)	(8,10)	(9,10)
(x_2, y_2)	(10,9)	(10,8)	(10,7)	(10,6)	(10,5)	(10,4)	(10,3)	(10,2)	(10,1)	(10,0)

Question 5.

What values can x_1 , x_2 , y_1 , y_2 take (write your answers in set notation)?

$$x_1 = 0, 1, 2, 3, 4, 5, 6, 7, 8 \text{ or } 9$$

$$y_1 = 10$$

$$x_2 = 10$$

$$y_2 = 9, 8, 7, 6, 5, 4, 3, 2, 1, \text{ or } 0$$

Question 6.

Press  > **New Document**, and then select **Add Calculator**. Then construct the following variable definitions. Your teacher will lead you through the steps to construct a single general equation which will generate a set of lines.

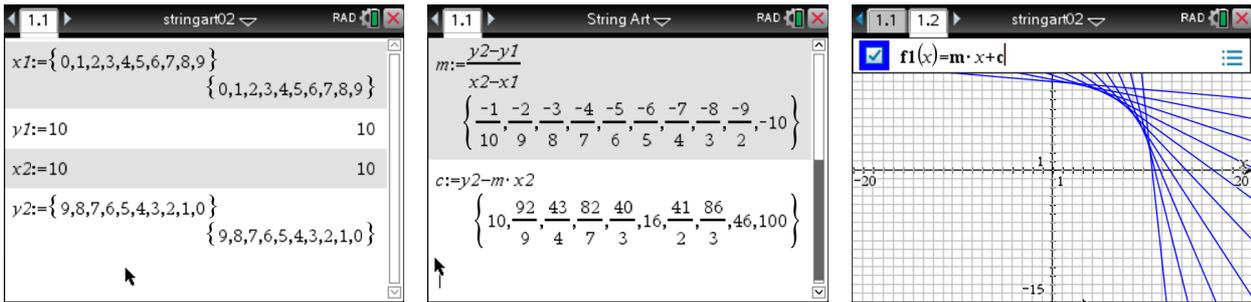
$$x1:=\{0,1,2,3,4,5,6,7,8,9\}$$

$$y1:=10$$

$$x2:=10$$

$$y2:=\{9,8,7,6,5,4,3,2,1,0\}$$

Teacher led discussion is best here, to help all students get the graph constructed as shown. Note that students will need to add a graph page with the same attributes as before.

**Step 5: Transforming the lines (teacher led)**

Now look at the constructed graphs – how are the parts of the designs that are in the remaining quadrants different to the first quadrant designs?

Question 7.

Observe your manually constructed design from Question 1. Consider the first quadrant line with

$$\text{equation } y = \frac{-1}{10}x + 10. \text{ What is the 'mirror image line' in the second quadrant? What are the}$$

coordinates used to construct the line, and how are they different from those used in the first

quadrant line? Try to create 3 new general equations in $f2(x)$, $f3(x)$ and $f4(x)$ that would complete the 'circle' design.

[This is probably best attempted as teacher led discussion about the transformation of ordinates, and how to modify the general equation to reproduce the design in the other quadrants. Sample screens shown.]

Quadrant 2:

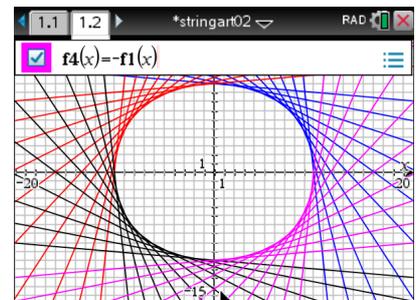
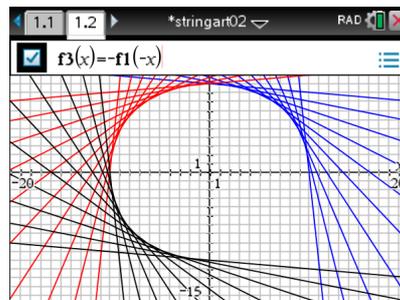
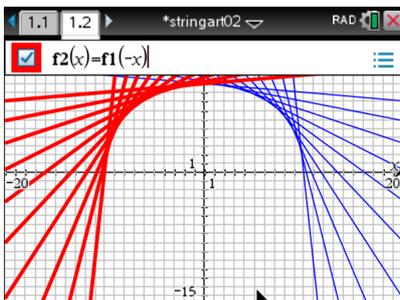
$$x \rightarrow -x : y \rightarrow y, \text{ so } f2(x) = f1(-x)$$

Quadrant 3:

$$x \rightarrow -x : y \rightarrow -y, \text{ so } f3(x) = -f1(-x)$$

Quadrant 4:

$$x \rightarrow x : y \rightarrow -y, \text{ so } f4(x) = -f1(x)$$



Step 6: Extension explorations: How can we change the design?

In this open-ended part of the investigation, explore one or more of the following challenging questions!

Question 8.

Identify the shape that forms the boundary of these 4 sets of straight lines. Is it really a circle? Is it a regular shape?

It is not really a circle – if you zoom in you will see that it has straight edge sides, so is a polygon. It can also be shown that the side lengths are not identical in length, and so it is not a regular polygon.

Question 9.

Modify your design to make a regular octagon.

Answers may vary, but here is one method.

$x1:=\{0,5\}$; $y1:=10$; $x2:=10$; $y2:=\{5,0\}$ and then re-enter the definition for m and c

Question 10.

Make a string art design with a similar shape to the original design, but much smaller?

Answers may vary, but here is one method.

$x1:=\{0,1,2,3,4\}$; $y1:=5$; $x2:=5$; $y2:=\{4,3,2,1,0\}$ and then re-enter the definition for m and c

Question 11.

Make a string art design that is more elliptical (hint: think how you could change the gradients but keeping the same y-intercept)?

Answers may vary, but here is one method.

In the graph page, alter the rule so that $f1(x)=2mx+c$ or $f1(x)=0.5mx+c$

Question 12.

Move the original design so it is centred at (2,0)?

[This is very hard for most students – even at Year 12!].

In the graph page, alter the general equation so that $f1(x)=m(x-2)+c$ (which sorts out the quadrant 1 and quadrant 4 graphs). However because of the horizontal reflections that produce the new quadrant 2 and quadrant 3 graphs, a translation for these will need to be added.

So $f2(x)=f1(-(x-4))$ and $f3(x)=-f1(-(x-4))$

Teacher notes

- This task arose from a classroom discussion relating to co-ordinate geometry. I was seeking to revise the idea of constructing the equation of a line from a pair of points, but decided to try a more ambitious exploration of the power of parameters and functions using technology.
- It would be beneficial to show a youtube video of the original idea with nails, cotton and hammers. Here's one example [<https://www.youtube.com/watch?v=y0XT1u-ygrs>]
- To begin the task, it is a good idea to get students to first create the pattern manually – this is important for revising the required by-hand skills, and to help them 'see' the patterns in the lines.
- Encourage students to calculate the gradients and y-intercept of each line from the first quadrant, and check the graphs of the equations with technology – this order seems useful for helping discover what benefits the technology has in reducing tedium, provided that they understand the underlying maths.
- The transformation of the graph of $f_1(x)$ to the other 3 quadrants can be a stretch for some students – focussing on the transformed points used seems to help, but some students find the function language " $-f_1(-x)$ " a bit confusing – I find going slow here and using the original diagram very useful at this stage of the task.
- Some of the extensions are very hard – use your judgement about whether students are getting frustrated. It is useful to try these yourself before asking students to attempt them.