Combinations

ID:8433

Time required
35 minutes

Activity Overview

This activity introduces students to combinations. They derive the formula for the number of combinations of n objects taken r at a time by starting with a list of permutations and eliminating those that name the same group, just in a different order. From here they see how the number of combinations is related to the number of permutations. Students then solve several problems involving combinations, including problems that involve geometry.

Topic: Permutations, Combinations & Probability

- *Use factorial notation to express the number of permutations and combinations of n elements taken r at a time.*
- *Evaluate expressions involving factorials to compute the number of outcomes in a sample space.*

Teacher Preparation and Notes

- *This activity is designed to be used in an Algebra 2 classroom. It can also be used in an introductory Statistics course or an advanced Algebra 1 course.*
- *This activity assumes that students have previous knowledge of finding permutations.*
- *Many students struggle with permutations and combinations. For this reason, it is important that you do not rush through the activity or assume that any part of the activity is "too easy." A solid foundation on these basic concepts will help them later when the problems become more complex.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "8433" in the keyword search box.***

Associated Materials

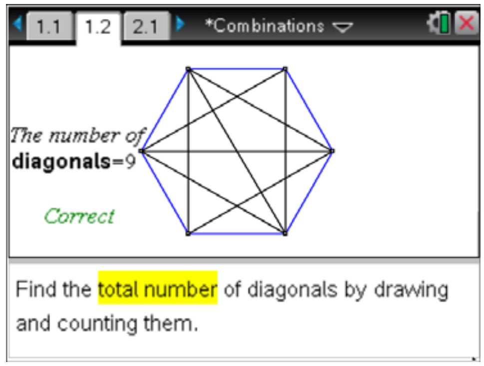
- *Combinations_Student.doc*
- *Combinations.tns*
- *Combinations_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Permutations & Combinations (TI-Nspire technology) — 12602*
- *What's Your Combination (TI-84 Plus family) — 10126*
- *Perms and Combs? (TI-Nspire technology) — 12645*

Problem 1 – An introduction

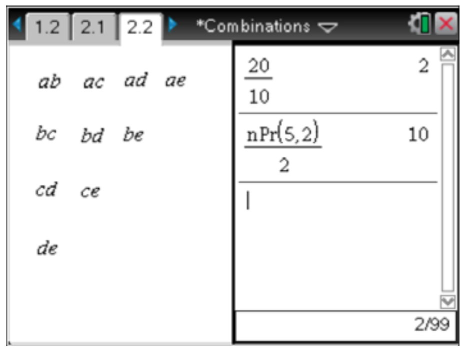


Have students move to page 1.2 and find the number of diagonals in a hexagon by using the **Segment** tool from the Points & Lines menu. If needed, remind them that a diagonal is a segment joining two nonadjacent vertices. (Students should find a total of nine diagonals).

Have students enter their amount by clicking twice on **diagonals=1** and entering their amount.

Later in this activity, students will answer the same question using *combinations*.

Problem 2 – Combinations

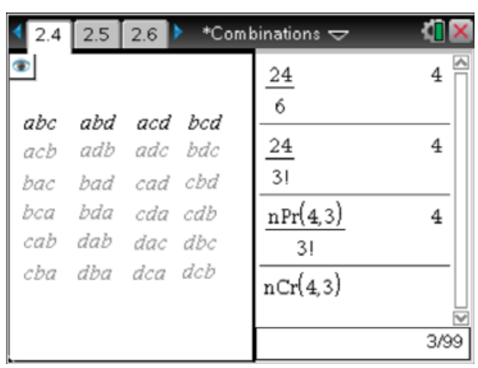


Students should read page 2.1 and advance to page 2.2, where they will reduce the displayed permutations to combinations. Explain that when finding combinations, **ab** is the same as **ba**; the order does not matter—both name the same group. They should select **Hide/Show** from the Tools menu and click on each arrangement to be hidden. Once all the desired permutations have been hidden, pressing the esc key closes out of **Hide/Show**.

Ask students how the number of combinations compares to the number of permutations. They should find that there are half as many combinations as permutations. In this instance, the number of combinations is 20 ÷ 2. Students should write a fraction using permutation notation (i.e., ${}_nP_r$) to represent the number of combinations. The division template can be accessed by ctrl + ÷.

Show that $\frac{20}{2} = \frac{{}_5P_2}{2}$ and ask what the number in the denominator might represent.

Students may conjecture that it is equal to *r*. They will check this conjecture by moving to pages 2.3 and 2.4.



On page 2.4, students should find 24 permutations, but only 4 combinations. Have them write a mathematical expression that uses permutations to show the number of combinations. They should write $\frac{{}_4P_3}{6}$. Ask why it makes sense that the number of combinations is the number of permutations divided by 6. (Students may wish to *show* the permutations they have hidden to answer this question.)

TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 1 at the end of this lesson.

Ask if their conjecture was correct. (No, because r = 3, and not 6). Tell students to write $\frac{{}_4P_3}{6}$ with 6 as a factorial $\left(\frac{{}_4P_3}{3!}\right)$ and have them make a new conjecture. The denominator is r! This was not necessarily apparent in the previous problem because 2! = 2.

On their worksheets, have students write $\frac{{}_4P_3}{3!}$ *without* permutation notation, and then replace the numbers 4 and 3 with r and n, respectively, to derive the formula for finding the number of combinations of n objects taken r at a time.

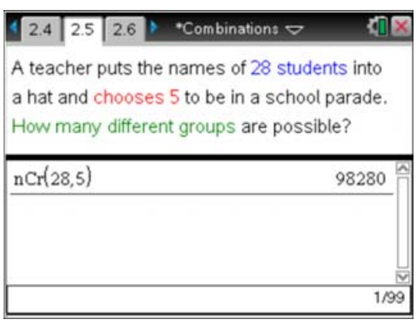
Note: You may need to remind students that ${}_nP_r = \frac{n!}{(n-r)!}$.

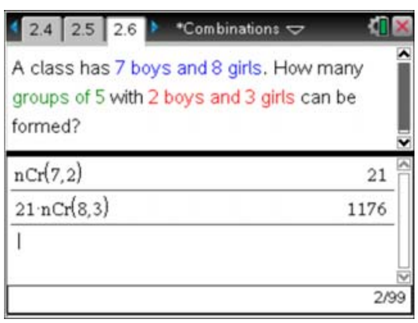
$${}_4C_3 = \frac{{}_4P_3}{3!} = \frac{4!}{(4-3)!3!} = \frac{4!}{1!3!} = \frac{n!}{(n-r)!r!}$$

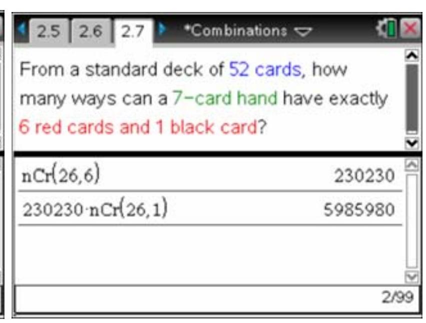
Students may use the *Calculator* application on page 2.4 to practice using this formula, or practice using the combination command, available in the Probability menu. As with finding permutations, list n and r in parentheses, separated by a comma.

Have students discuss how combinations differ from permutations. Stress that permutations are *arrangements* and combinations are *groupings*. Challenge them to think of scenarios that would use combinations rather than permutations.

Working independently, students should answer the questions on pages 2.5, 2.6, and 2.7. When all are finished, review the questions and answers and clarify any confusion the students might have. The question on page 2.5 is straightforward; the questions on pages 2.6 and 2.7 require more thought.





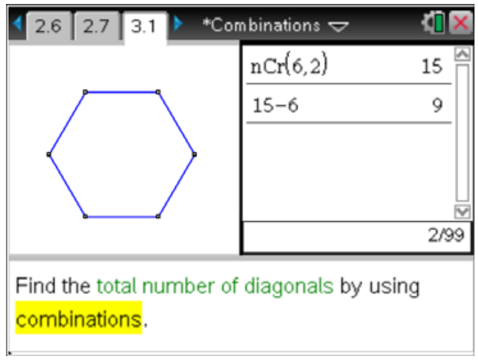


TI-Nspire Navigator Opportunity: *Screen Capture* and *Quick Poll*

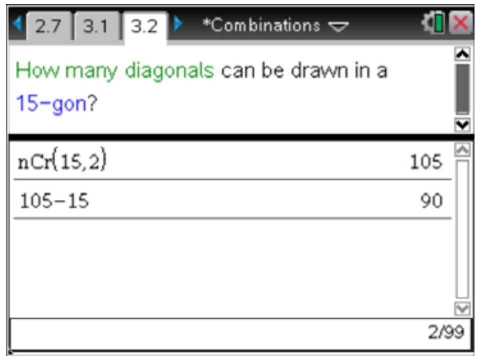
See Note 2 at the end of this lesson.

Problem 3 – Combinations and Geometry

On page 3.1, students are asked to answer the introductory question using combinations. Let students brainstorm collaboratively to discover why ${}_6C_2$ is not the correct answer—namely, ${}_6C_2$ gives the number of segments between ***any*** two points; the segments joining adjacent vertices must be subtracted from this total.)



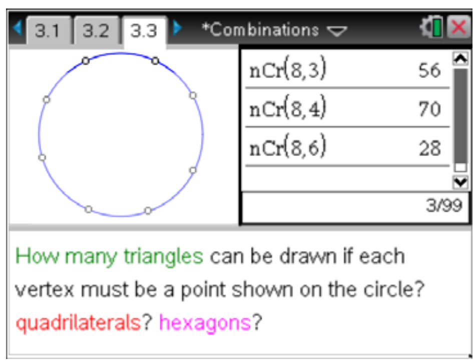
Students should extrapolate from the hexagon example to answer the question on page 3.2.



Let students work independently to answer the questions on page 3.3.

There can be 56 triangles, 70 quadrilaterals, and 28 hexagons.

Ask students why these questions involve finding combinations and not permutations.



Extension

The extension involves two critical thinking questions. Let students work on them independently.

Part 1: Give two different explanations for why ${}_nC_n$ is always equal to 1.

There is only one way to make a group that involves every possible choice.

Also, ${}_nC_n = \dfrac{n!}{(n-n)!n!} = \dfrac{n!}{0!n!} = \dfrac{n!}{1n!} = \dfrac{n!}{n!} = 1.$

Part 2: Find ${}_8C_2$, ${}_8C_6$, ${}_7C_3$, and ${}_7C_4$. Then determine a general rule.

${}_8C_2 = {}_8C_6 = 28;\ {}_7C_3 = {}_7C_4 = 35;$ In general, ${}_nC_r = {}_nC_{n-r}$.

TI-Nspire Navigator Opportunities

Note 1
Problem 2, *Live Presenter*

Use *Live Presenter* with pages 2.2 and 2.4 to demonstrate how to hide text as well as to help aide in the class discussion of the desired combinations.

Note 2
Problem 2, *Screen Capture and Quick Poll*

Use *Screen Capture* to monitor student progress as they work through the problems on pages 2.5 - 2.7. Send a *Quick Poll* for each question on pages 2.5 – 2.7. Use the *Quick Poll* results to check for student understanding - offering differentiated instruction where needed.