EXPLORATIONS

Name ______________________________

Date ______________________________

Activity 2

The Shrinking Dollar

Have you ever noticed how the costs of things you buy always seem to go up but rarely go down? If you purchased a pair of your favorite jeans for $30 last year, the cost this year is probably higher, and the cost next year will most likely be higher still. Much of the increase in cost is due to an economic factor called *inflation*.

In this activity, you will calculate changes in the cost of goods due to inflation.

The Problem

When economists report that the annual inflation rate is at 4%, they are predicting that the average cost of goods will increase about 4% over a year. If an item costs $5.00 today and inflation is 4% per year, the predicted cost in one year could be calculated as:

$5.00 + (4% of $5.00) = $5.00 + 0.04 x $5.00 = $5.00 + $0.20 = $5.20.

An alternative (and more efficient) way to calculate the increased cost is:

1.04 x $5 = $5.20

Either method shows that a $5.00 item would cost $5.20 after one year of 4% inflation.

How much would the $5.00 item cost after two years of 4% inflation?

In this activity, students examine the possible long-term effects of inflation. The compounding effect of inflation from year to year is one example of exponential growth. Other examples are provided in the ***Problems for Additional Exploration****.*

The second method of computation is more useful for calculator implementation. Be certain to have students explain why the two methods are equivalent.

Encourage students to use results of their completed work. The process for determining the cost after two years is the same as computing the increased cost of a $5.20 item after one year of 4% inflation; that is, 1.04 x $5.20 = $5.41. The number 1.04 is sometimes referred to as the ***inflationary factor****.*

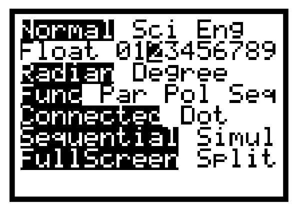
Using the Calculator — Part One

Your calculator makes it easy to investigate increases due to inflation. To determine the effects of 4% inflation on an item that currently costs $5.00, follow the instructions given in the left column below. The information at the right explains the purpose of the key sequences.

Calculating the Results

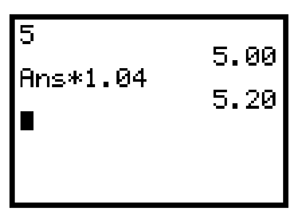
*Once the mode is set to "fix" two decimal places, the calculator will display all values on the home screen using that format. When they complete this activity, students should reset this mode to **Float**. Displays on the TI-80 and TI-83 differ slightly from the one shown for the TI-82, but a similar setting can be made.*

When you do this:	Your calculator does this:
1. Press [MODE].	*Displays the mode screen.*
2. Press [▼] (the down blue arrow key) and [▶] (the right blue arrow key) to move the cursor over the **2** in the line that begins with **Float**.	*Displays its answers correctly rounded to two-decimal places. Your calculator display should look something like this:*



When you do this:	Your calculator does this:
3. Press [ENTER].	*Returns to the home screen.*
4. Press [2nd] [QUIT] [CLEAR].	*Clears the display.*
5. Press 5 [ENTER].	*Enters the beginning value for the computations. In this case, the 5 represents the $5.00 initial cost.*
6. Press [×] 1 [.] 0 4 [ENTER]	*Multiplies the last value displayed (5.00) by 1.04 (the inflationary factor). Your display should look like this:*

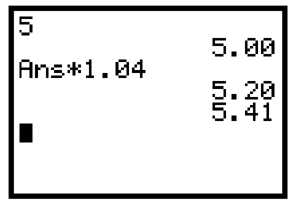
*Note that the calculator automatically displays **Ans** as soon as [×] is pressed. **Ans** is short for Answer, the last value displayed.*



The display shows the cost after one year of inflation to be $5.20, which agrees with our earlier computations.

7. Press [ENTER]

Repeats the last command (multiplies the last value displayed by 1.04). Since the last value displayed is 5.20, the display will show the results of calculating 5.20 x 1.04.



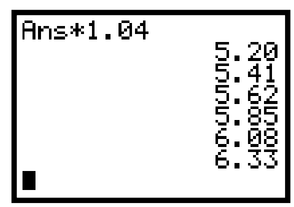
The value $5.41, the cost of a $5.00 item after two years of 4% inflation, should agree with your previous calculations.

8. Press [ENTER] four more times to determine the cost after six years.

Displays the increased cost for each additional year.

Once you enter the initial instruction into the calculator, pressing [ENTER] displays the increased cost due to compounding inflation for an additional year.

The final display should look like this:



- Use Table 2.1, **Cost of a $5.00 Item Due to 4% Inflation Compounded Yearly**, in the **Questions** section to record the results displayed on your calculator. Continue your calculations (continue pressing [ENTER]) to determine and record values for the remaining cells in the table.

- Answer #1 through #4 in the **Questions** section of this Activity.

Your calculator can display a graph of the increased costs as a function of time. To do so, you must first place your table entries into the calculator's **LIST** storage.

Displaying a Graph

The final value in the table should show that after eighteen years of 4% inflation, the cost of a $5.00 item has increased to $10.13, or approximately twice its initial cost.

1. Press [STAT] **4:ClrList**. Press [2nd] [L1] [,] [2nd] [L2] [,] [L3] [ENTER] to clear the needed lists.

   You will enter the information in Tables 1 and 2 in lists **L1** *and* **L2**. *You will use the third list,* **L3**, *to store additional values later in this activity.*

2. Press [STAT] **1:Edit**. Press [ENTER] to gain access to the calculator's list storage.

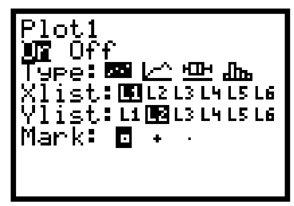
Yearly increases get larger as the number of years of inflation increases. You will want the students to realize that this result is due to the compounding effect of inflation.

3. Type the numbers from the column labeled **Year** into **L1** (the first list) on your calculator. Type a number, press [ENTER], and repeat until all numbers have been entered.

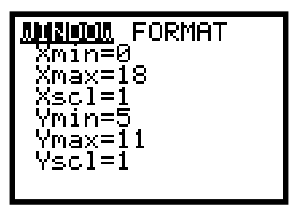
4. Press [▶] to move to the second list. Type the numbers from the **Cost** column into **L2** (the second list) on your calculator.

The 4% increase is computed on values that get larger each year.

5. To see a scatterplot graph that displays the increasing costs, press [2nd] [STAT PLOT] [ENTER]. Edit the window so that it looks like the one at the right. To highlight a selection, use the blue arrow keys to move the blinking cursor over the desired location and press [ENTER]. By selecting the first option for **Type** of graph, you are selecting a scatterplot.



6. Press [WINDOW] and edit the numbers until your screen looks like the one at the right. Why do you suppose these window settings were selected?



7. Press [GRAPH] to view a scatterplot of the increasing costs as a function of the number of years of inflation.

8. To view the coordinates of points on this plot, press [TRACE] and use [▶] (the right blue arrow key) to highlight different points.

✐ Answer #5 and #6 in the **Questions** section of this activity.

Using the Calculator — Part Two

You have investigated the effects of 4% inflation. Now let's investigate the situation where the inflation rate is twice as high—at 8%.

Calculating the Results

- Complete Table 2.2, **Cost of a $5.00 Item Due to 8% Inflation Compounded Yearly**, using the same procedures used for Table 2.1. (Begin by pressing [CLEAR] 5 [ENTER] [×] 1 [.] 0 8 [ENTER] [ENTER] [ENTER] and so on.)
- Answer #7 and #8 in the **Questions** section.

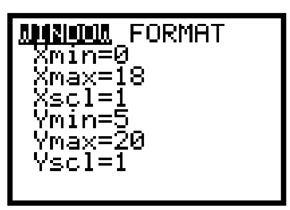
Displaying a Graph

You can graphically compare the effects of 4% and 8% inflation.

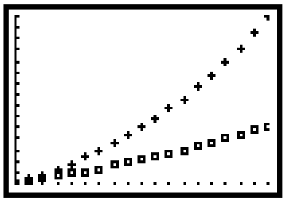
The increasing nature of the yearly increases is somewhat subtle. If you move from left to right, the size of the vertical jump gets larger each time. You can observe this by noting the relationship between the top and bottom edges of consecutive "points." If the increases were constant, all points on the plot would appear to fall on a straight line rather than on a curve.

1. Enter the values from the **Cost** column of Table 2.2 into list **L3** of your calculator.
2. Press [2nd] [STAT PLOT] 2 to set up a second scatterplot (using **Plot2** of the three available in the **Stat Plot** menu).
3. Edit the window so that yours looks like the one at the right. Recall that to make a selection, you will use the blue arrow keys to move the blinking cursor on top of the desired location, and then press [ENTER].

4. Since the costs associated with the 8% inflation are larger than those associated with the 4% inflation, you will need to adjust the viewing window for the graph. Press [WINDOW]. One possible window is defined by the settings shown at the right.

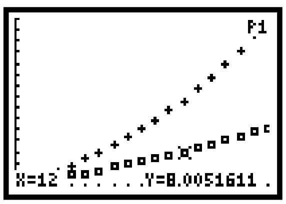


The dual scatterplot should look like this:

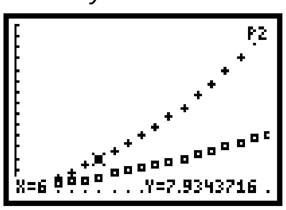


The nonlinear relationship between time and cost is made more obvious by these plots.

For 4% inflation, it takes about 12 years:



For 8% inflation, it takes about 6 years:



5. When you have changed the window settings, press [GRAPH] to view both scatterplots on the same screen.

6. To display coordinates of points on either graph, press [▲] to select either **P1** (the 4% inflation plot) or **P2** (the 8% inflation plot). You will know which plot is selected by the upper right corner of the display window.

7. As before, pressing [TRACE] and [▶] or [◀] permits you to move from one point to another on the selected plot. Trace the plots to determine the number of years it takes for the $5.00 item to increase to a cost of $8.00 at 4% inflation and then at 8% inflation.

✎ Answer #9 in the **Questions** section.

Questions

Table 2.1

Cost of a $5.00 item due to 4% inflation compounded yearly

Year	Cost
0	$5.00
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	

Table 2.2

Cost of a $5.00 item due to 8% inflation compounded yearly

Year	Cost
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	

The final value in the table should show that after 18 years of 4% inflation, the cost of a $5.00 item has increased to $10.13, or approximately twice its initial cost.

Year 2 cost minus Year 1 cost is $0.21.

1. About how many years does it take for the initial $5.00 cost to *double* due to inflation?

2. The increase in cost after one year of inflation is 20 cents ($5.20 - $5.00). What is the increase in cost from Year 1 to Year 2?

Because of the compound effects of inflation, yearly increases get larger as the number of years of inflation increases.

3. What happens to the yearly increase as the number of years of inflation increases?

The 4% increase is computed on values that get larger each year.

4. Why do the yearly increases get larger with time even though the inflation rate remains constant at 4%?

The graph rises from left to right, indicating that as the number of years increases, the item costs (represented by the plotted points) also increase.

Return to page 12, ***Displaying a graph****.*

5. What feature of the graph illustrates that the cost of the $5.00 item is increasing each year?

The expanding nature of the yearly increases is somewhat subtle. If you move from left to right, the size of the vertical jump gets larger each time. If the increases were constant, all points on the plot would appear to fall on a straight line rather than on a curve.

6. How does the graph for Table 2.2 show that the yearly increases are getting larger as time passes?

Return to page 13, ***Using the Calculator — Part Two****.*

7. How long did it take for an item to double in cost at 8% inflation?

After nine years of 8% inflation, the cost of the item has doubled to $10.

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8. How many times more costly was the item after eighteen years of 8% inflation than at year 0?

After 18 years of 8% inflation, the cost of the item has nearly quadrupled to $19.98.

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Return to page 13, ***Displaying a graph.***

9. Why do you think this activity was called *The Shrinking Dollar*?

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Problems for Additional Exploration

Students will rarely find the exact $10 figure in their computations. They should discuss which year to record—the last one (to show an amount less than $10) or the first one (to show an amount greater than $10).

1. You have seen that at 4% inflation, it takes approximately eighteen years for the cost of a $5.00 item to double. At 8% inflation, doubling the cost took approximately nine years.

   Use your calculator to help you complete the following table, which shows the approximate number of years it takes for an item to double in cost at various inflation rates.

Inflation Rate	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
Years to Double Cost		18				9				

*If they select the year that gives a result closest to $10, they will have an easier time seeing the approximate relationship described by the rule of 72: the product of the inflation rate R and the years Y needed to double the cost is approximately 72; that is, R*Y≈72.*

Students might be asked to predict how many years it would take for the cost to double under 18% inflation. They could then test their prediction using the calculator.

a. Can you describe an algebraic relationship between the inflation rate R and the number of years Y needed for the cost to double?

Hint: *The relationship between R and Y is sometimes called the Rule of 72.*

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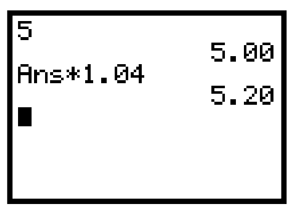
b. Construct a scatterplot of your results.



2. Inflation is a constant problem to all of us since it eats away at our buying power. The long-term effects of inflation are difficult to predict but if past experience is a guide, you can expect significantly higher prices by the time you are an adult and even higher prices by the time you are ready to retire. The following problems illustrate some of the concerns that you may face in the future.

If one assumed that the current cost of a candy bar is $0.50, then after 50 years of 5% inflation, that same candy bar would cost approximately $5.73.

a. Assume that it is fifty years from now and you, now a grandparent, are visiting your grand-children. To keep them happy you offer to give each of them the money to buy a candy bar. Inflation has been running at 5% every year for the past fifty years. How much do you think you will need for each of those candy bars?

Make an estimate and then use the calculator to compute a more exact answer.

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b. Using the situation in question **2a**, assume that the inflation rate has been 10% every year for the past fifty years. Now how much do those candy bars cost? Make an estimate and then use the calculator to compute a more exact answer.

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Students may assume that the cost would be twice its original cost because the inflation rate has doubled. However, the cost of a $0.50 candy bar after fifty years of 10% inflation is approximately $58.70, more than ten times as large!

3. Interest charged on credit card accounts, as well as interest paid on deposit accounts at banks and savings institutions, grows in ways very similar to inflation. One major difference might be in the period of time between compounding (those times when interest is computed and either added to the fund or charged to the account). Where the inflation examples assumed compounding once each year, interest may be calculated each quarter (that is, every three months), each month, or even each day.

a. Assume that you have deposited $1000 into a savings account. If the annual interest rate on a savings account is paid at 6% compounded annually, how much money will be in the account at the end of five years?

At the end of five years, the $1000 will have grown to approximately $1338.23.

Calculator solution:

Press [CLEAR] 1 0 0 0 [ENTER] [×] 1 [.] 0 6 [ENTER] [ENTER] [ENTER] and so on.

Students will need to press [ENTER] sixty times to produce the final amount on deposit: $1348.85.

b. If the $1000 were deposited into an account that paid 6% annual interest compounded every *month*, the monthly interest rate would be 6% ÷ 12 or 0.5% per month. If the money were left on deposit for five years, interest would be paid sixty times (once for each of the sixty months in the five-year period). How much money would be in the account at the end of the five years under this scheme?

Note that the amount resulting from monthly compounding is only about $10 more than that from annual compounding.
If your students have studied exponents, you may want to develop a "closed formula" approach to this problem: $1000(1+.06/12)^(5*12)= $1348.85.*

Calculator solution:

Press [CLEAR] 1 0 0 0 [ENTER] [×] 1 [.] 0 0 5 [ENTER] [ENTER] [ENTER] [ENTER] and so on.

How does this final amount compare with the final amount using *annual* compounding?

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You could then have them determine the amount that would result from daily compounding. Note that it is not much larger:

$1000*(1+.06/365)^(5*365)=

$1349.83

c. Assume that you have made a charge of $1000 on a credit card that charges 22.5% annual interest compounded monthly. If you were to leave this charge on the card for a period of five years and make no further charges or payments, how much would you owe the credit card company? Make an estimate first, then use the calculator to compute a more exact value.

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*For the credit card problem, use a monthly rate of 22.5% ÷ 12 or 1.875%. Using **Ans** * 1.01875 sixty times gives a final amount of $3048.30 (more than triple the amount charged). This might serve as motivation for a discussion of the benefits of paying credit card debts as quickly as possible. Compare these results with the savings balance in part **b** of this problem.*