NUMB3RS Activity: Fresh Air and Parabolas
Episode: “Pandora’s Box”

Topic: Quadratic functions, trajectories, vectors, parametric functions **Grade Level:** 10 - 12
Objective: Students will investigate linear and quadratic relationships to solve problems both symbolically and graphically.
Time: 15 - 20 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator

Introduction

In “Pandora’s Box,” a corporate jet crashes in a forest. The crash is first thought to be an accident, but the cause becomes more suspicious as Charlie studies the crash scene. The airplane’s flight management computer (FMC) contains important information about the conditions of the plane moments before the crash and should aid in determining the cause of the crash. The clean up crew is unable to find the FMC as they search the debris field. Charlie and Amita begin to search for the FMC by marking the location of parts of the wreckage with flags and measuring the distance from the flags to the point of impact. Charlie states that there is “nothing like fresh air and the geometry of predictive trajectories” as he and Amita try to determine the location of the missing FMC. In this activity, students will study equations for trajectories using linear and quadratic functions. Students will use parametric functions to display the trajectories on their graphing calculators.

Discuss with Students

Students should understand that the trajectory in Question 1 should be a parabola opening downwards. You may want to see what they draw and have a discussion about the path of the ball. You may want to use an actual ball to allow students to see the actual trajectory. The downward path of the ball is shown in the illustration to be a straight line. The path of an actual ball will not be straight but slightly curved downward. This illustration was done to simplify the problem—a linear approximation is accurate for short distances.

Question 2 is designed for students who have had some experience with quadratic functions of the form $y = ax^2 + bx + c$. You may want to encourage students to use their graphing calculators to make and test possible functions. Be sure to emphasize the importance of making an appropriate window on their graphing calculator as they make their conjectures. They should notice that **Xmin** should be around 0, **Xmax** should be around 250, and **Ymin** should be around 0 before they begin graphing. They can use the table function to determine appropriate values for **Ymax**.

Textbooks often use functions of the form $h(t) = at^2 + bt + c$ to describe the height of a ball. This function describes the relationship between the height of the ball and time. This function is a parabola but it is not the same parabola as the path of the ball. Students often confuse the graph of a relationship between distance and time with the actual path of an object. The path of the ball will be described using a vector function. You may want to discuss how this vector function describes the horizontal and vertical position of an object over time and how the parametric graph represents the actual path of the object. You may want to help students set up the parametric mode on the calculator, explain that in this case T represents time, and discuss how to determine appropriate window settings and T steps.

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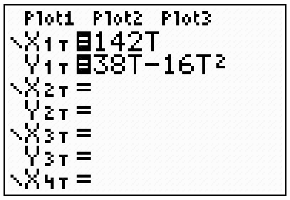
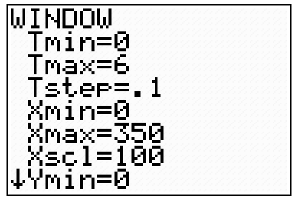
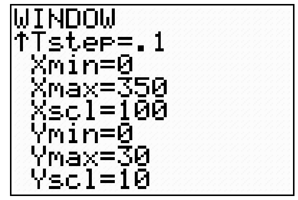
The equations and window settings used to display the trajectory as needed for Question 4 are shown below:







Student Page Answers:
1. *Answers will vary. The trajectory should resemble a parabola opening downward.* **2.** *Answers will vary. All quadratic functions that satisfy the conditions have* $a < 0$, $c = 0$, *and* $b/a = -250$.
3. $v_x \approx 142$ *feet/second and* $v_y \approx 38$ *feet/second* **4.** *The nose cone will be approximately 338 feet from the point of impact and it will spend approximately 2.38 seconds in the air.* **5.** *The speed of the jet is approximately 130 feet per second or 89 miles per hour.*

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Name: ________________________________ Date: _______________

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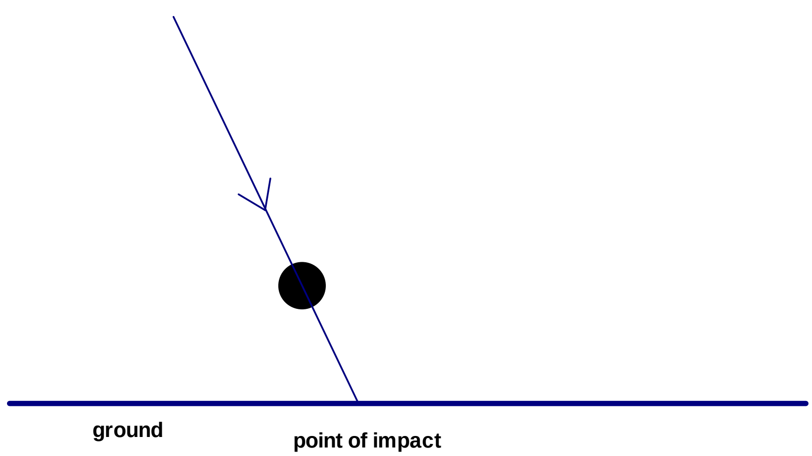
Trajectories

When the jet crashed, it broke into many pieces. Some of the pieces bounced off the ground and traced a path before they came to rest on the ground.

1. Imagine that a ball is thrown down at an angle as shown in the picture below. Draw a possible path the ball might travel after it hits the ground at the point of impact until it hits the ground again.



The path you just drew is called the *trajectory* of the ball. There are many different types of mathematical equations used to describe the paths of these trajectories. We will use quadratic functions and vectors to study trajectories in this activity.

Quadratic Functions

For this activity, assume that the only force acting on a piece of wreckage thrown from the jet is gravity, and that the effect of wind or air resistance is negligible. The trajectory of a piece of wreckage under these assumptions will be a *parabola* also known as a *quadratic function*.

The standard form of a quadratic function is $y = ax^2 + bx + c$, where y is the dependent variable, x is the independent variable, and a, b, and c are parameters that control the size and location of the parabola.

2. Suppose that Charlie and Amita find the engine of the airplane 250 feet from the point of impact. If the point of impact is the origin (0, 0), and the ground is flat (i.e., has a slope of 0), then the coordinates of the engine are (250, 0). There are many quadratic functions that pass through these two points and provide positive y-values for x-values between 0 and 250. Find at least three different quadratic functions that satisfy these conditions. Explain what is true with the parameters a, b, and c for all quadratic functions that satisfy these conditions. Use your graphing calculator to help you find the function rules.

Using Vectors

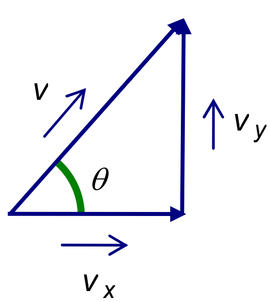
When Charlie does his calculations in his notebook, he uses vectors to describe the trajectories. The position of a piece of wreckage at time t seconds after the crash can be expressed as a vector. The first coordinate describes the horizontal position of the object at time t, and the second coordinate describes the vertical position.

$$\textbf{Position Vector Function: } p(t) = \left(v_x \bullet t,\ v_y \bullet t - \frac{g \bullet t^2}{2} \right)$$

$p(t)$ stands for the position of the object at time t given as an (x, y) coordinate
t stands for the number of seconds after initial impact
v_x is the initial horizontal velocity (feet/second)
v_y is the initial vertical velocity (feet/second)
g is the acceleration due to gravity (32 feet/second^2)

Suppose the nose cone of the jet is thrown into the air at a speed of 100 miles per hour (about 147 feet per second). The initial horizontal and vertical velocities of the nose cone will be different depending on the angle that the jet crashes into the ground. Assume this angle to be 15° from the ground. The following formulas can be used to find the initial horizontal and vertical velocities.

$v_x = v \bullet \cos(\theta)$
$v_y = v \bullet \sin(\theta)$



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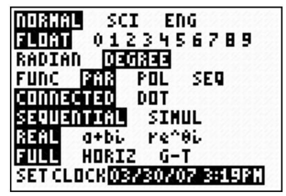
3. Use the formulas above to calculate the initial horizontal and vertical velocities of the nose cone.

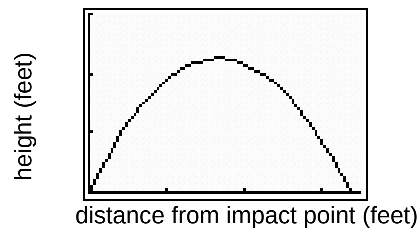
Substitute your answers from Question 3 and g = 32 feet/second^2 into the position vector function:

$$p(t) = \left(v_x \bullet t,\ v_y \bullet t - \frac{g \bullet t^2}{2} \right)$$

This vector function can be entered into your calculator using the parametric mode. To change your calculator to parametric mode, press MODE and use the settings shown to the right.



Press Y= and enter the equations for the trajectories. Press GRAPH to see the trajectory. Be sure to adjust your window settings to see the entire trajectory. **Tmin** represents the beginning time for the trajectory, so this amount should be 0 seconds. **Tmax** is the number of seconds when the trajectory ends. You should set this amount large enough to see the object hit the ground. **Tstep** should be 0.1 seconds.



4. Press TRACE and use the arrow keys on your calculator to approximate distance from the impact point where the nose cone comes back to ground. Also determine the number of seconds the nose cone was in the air. Use the position vector function to verify these results algebraically.

5. Suppose Charlie and Amita locate a piece of wreckage 457 feet from the point of impact. An eyewitness estimates that the object was initially thrown into the air at a 30° angle from the ground. Determine the initial velocity of the object as it was thrown into the air. Make the same assumptions as before that the only force acting on a piece of wreckage thrown from the jet is gravity, the ground is flat (has a slope of 0), and the effect of wind or air resistance is negligible.

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Trajectories with Other Assumptions

- Find the trajectory formula for wreckage of a jet traveling 500 miles per hour impacting at an angle of 10° from the ground. Make the same assumptions as you did in the activity except that the ground slopes down from the impact point at 8°.

- Investigate how the trajectory formula would be altered if wind was blowing directly against the horizontal path of a piece of wreckage at 15 feet per second.

- In this activity, several assumptions were made about how all of the debris from a plane crash would be dispersed similarly. In reality, the wreckage from a crash would be spread out over a large area. Identify some specific variables that would cause the debris field to be so large, and explain why this would happen.

Related Web Sites

- Investigate and make connections among various representations of quadratic functions using dynamic Java applets at **http://mathforum.org/te/alejandre/four/parabola.html**.

- An interactive applet showing the relationship between the position of an object and a graph over time can be found at **http://www.mste.uiuc.edu/murphy/MovingMan/MovingMan.html**.

- A lesson written to investigate the relationship between the angle a hose is held and the vertical distance of the water can be found at **http://www.col-ed.org/cur/math/math28.txt**.

- Investigate the relationships among various characteristics of a square using a dynamic graphing tool at **http://illuminations.nctm.org/ActivityDetail.aspx?ID=121**.

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