Properties of Logarithms

Student Worksheet

Name ______________________

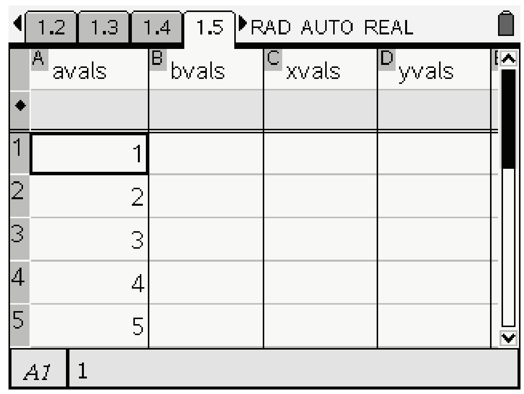
Class ______________________

Problem Statement

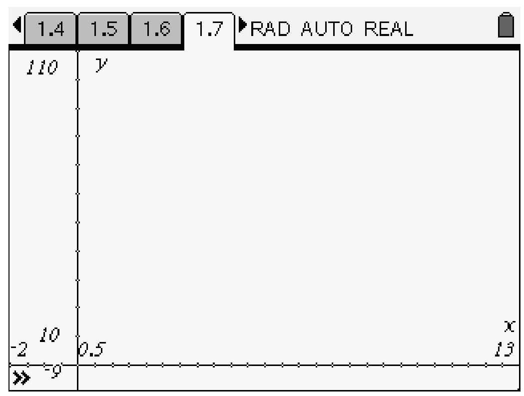
As you have probably noticed by now, in algebra it is often useful to write expressions in different forms so that we can more easily notice properties or behaviors of these expressions. Logarithms are no different. Suppose you wanted to write the logarithm of a power like $\log(a^2)$ without the use of the power. How might you do it?

1. We will begin investigating this question by defining a new variable $b = a^2$.

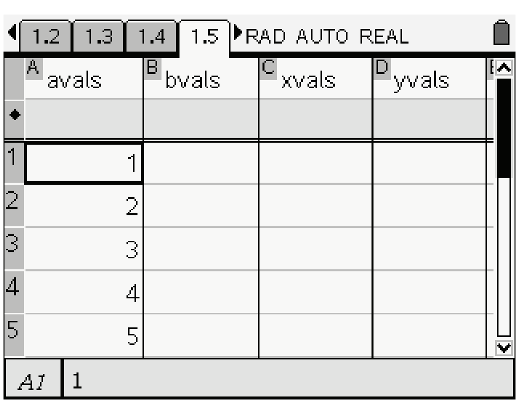
a. Enter a formula in Column B on page 1.5 of the TI-Nspire document *CollegeAlg_PropsOfLogs.tns* to calculate *b* for the given values of *a*. Record some of your values on the screen to the right.

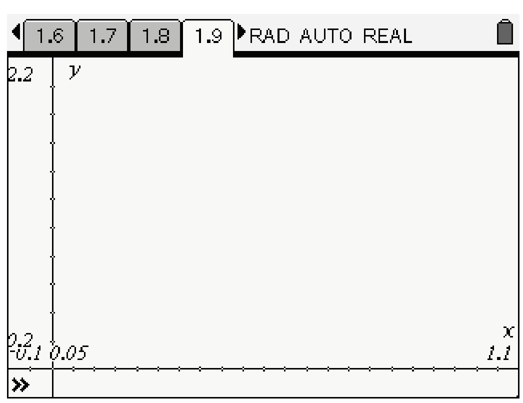


b. Plot **avals** vs. **bvals** as a scatter plot on page 1.7 of the TI-Nspire document and sketch your graph on the axes to the right. Describe the shape of the graph. Is this what you expected? Explain your thoughts below.



c. Now we will define *x* and *y*. Enter formulas in Columns C and D of page 1.5 so that $x = \log(a)$ and $y = \log(b)$. Record some of your values from the spreadsheet on the screen below. Then plot **xvals** vs. **yvals** as a scatter plot on page 1.9 of the TI-Nspire document and make a sketch of your graph below.



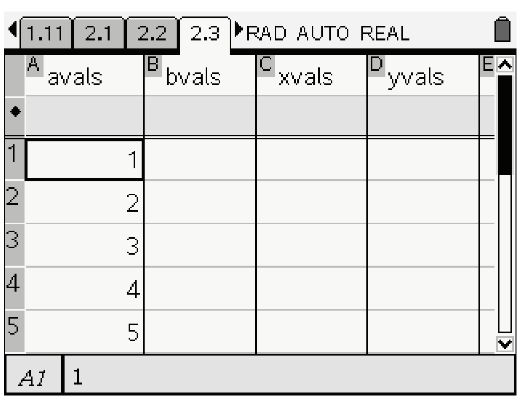


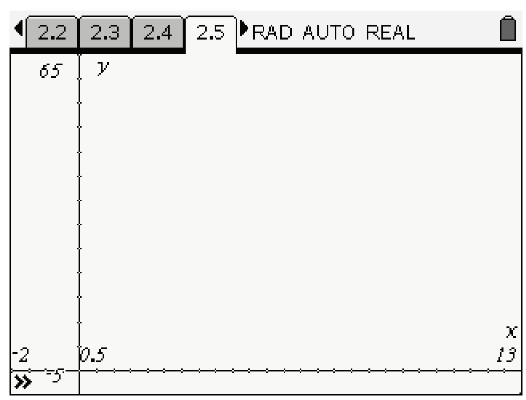
d. Describe the shape of the graph. Is this what you expected? Explain your thoughts below. On your *Graphs & Geometry* page, use the **Line** tool to draw a line through the points and sketch it above. Use the **Coordinates and Equations** tool to find the equation of the line and record your equation.

e. We can substitute back the original expressions we used for x and y to rewrite this as an equation in terms of a and b. Use the equation of the line you found and make the substitutions $x = \log(a)$ and $y = \log(b)$ to get a relationship between a and b. State this property formally as the *power property of logarithms* for the general log expression $\log(a^n)$.

2. At this point, we have examined the relationship for taking the logarithm of a variable raised to a power, but what about other arithmetic operations and their relationship with logarithms? As before, let's define a new variable $b = 6a$.

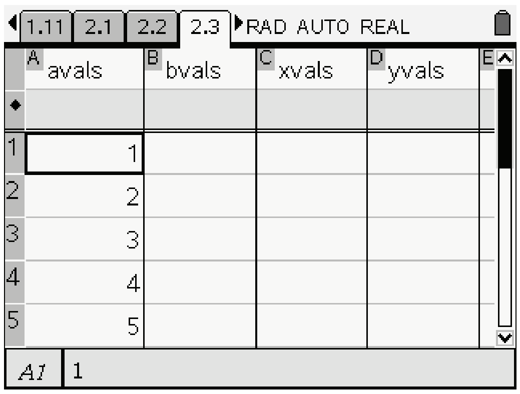
a. Enter a formula in Column B on page 2.3 of the TI-Nspire document to calculate b for the given values of a. Record some of your values on the screen below. Now plot **xvals** vs. **yvals** as a scatter plot on page 2.5 of the TI-Nspire document and make a sketch of your graph below.

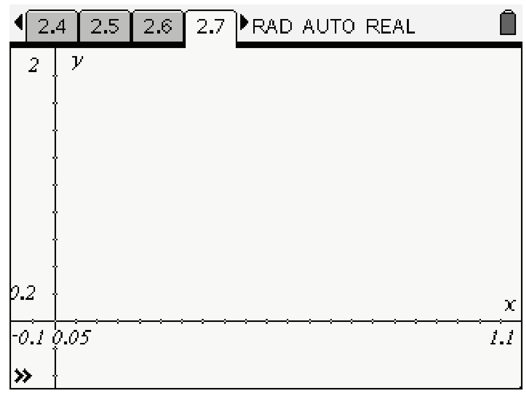




b. Describe the shape of the graph. As before, draw a line through the data and show the equation of the line. What is its slope and y-intercept? Is this what you expected? Explain your thoughts below.

c. Now define x and y. Enter formulas in Columns C and D of page 2.3 of the TI-Nspire document so that $x = \log(a)$ and $y = \log(b)$. Record some of your values from the spreadsheet on the screen below. Then plot **xvals** vs. **yvals** as a scatter plot on page 2.7 of the TI-Nspire document and make a sketch of your graph below.

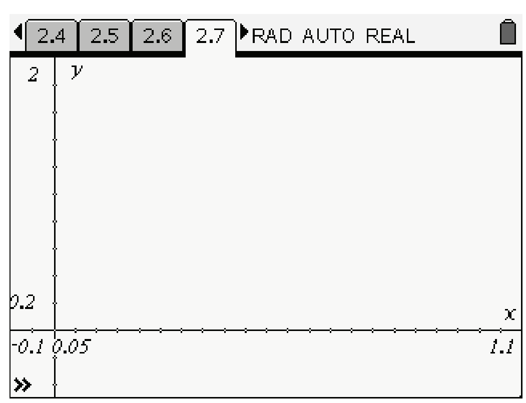




d. Describe the shape of the graph. As before, draw a line through the data and show the equation of the line. Use the **Graph Trace** tool to find the coordinates of the y-intercept to 4 or more decimal places. Is this what you expected? Where did this y-intercept come from? Here's a hint: Calculate $10^{0.778151}$. What is 0.778151? Explain your thoughts below.

e. Using the linear equation you found in x and y, substitute $x = \log(a)$ and $y = \log(b)$ into your equation to rewrite this as an equation in terms of a. This equation is an example of the *product property of algorithms*. In general, for $a > 0$ and $b > 0$, state this property for $\log(a \cdot b)$ based on your observations from this investigation.

3. In Problems 1 and 2, you used substitution to identify two properties of logarithms. In particular, you let $x = \log(a)$ and $y = \log(b)$. There is a special kind of graph paper called log-log paper that lets you make these substitutions graphically, by hand. Page 3.3 of the TI-Nspire document is an example of log-log paper. Examine the graphing page—a copy of the page is given below for your reference. Log-log paper is made by taking the logarithm of each value of x and y and then renumbering the axes.

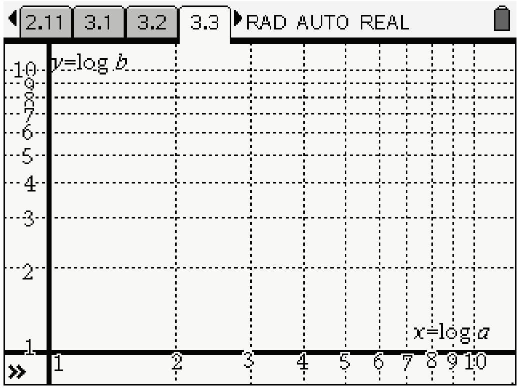


a. Why do the axes start at 1 instead of 0? Explain your thoughts below.

Note that spaces between the lines are not equal. That is because the distance from 1 to 2 on a logarithmic scale is equal to log 2 = 0.30102999566398 unit, and the distance from 1 to 3 is equal to log 3 = 0.47712125471967 unit, and so on. To illustrate how this graphical representation works, we will use the log-log paper to simplify the expression $\log\left(\frac{8}{a}\right)$.

b. Define $y = \frac{8}{a}$. Then use the **Point** tool to graph the function $y = \frac{8}{x}$ on the log-log paper by calculating values and placing the points on the grid. Graph only points with whole-number coordinates. What is the shape of the graph? What is its slope? Record your findings below.

c. As before, draw a line through the points. Use the **Coordinates and Equations** tool to find the equation of the line. Sketch your results on the axes below. What is the *y*-intercept of the line? Use the **Graph Trace** tool to find the coordinates of the *y*-intercept to 4 or more decimal places. Explain your thoughts below.



d. Where did this *y*-intercept come from? Can you guess? Use the *Calculator* application below to check. Explain how 0.90309 is related to this situation.

e. Use the fact that on your graph, $y = \log\left(\frac{8}{a}\right)$ and $x = \log(a)$ to substitute into the linear equation you found. Now rewrite this as an equation in terms of *a*. This can be referred to as the *quotient property of logarithms*. In general, for $a > 0$ and $b > 0$, state the property below by finishing the equation and explaining your reasoning.

$\log\left(\frac{b}{a}\right)$