# Crawling Around:
Parametric Plots

Imagine you are observing a bug crawling around in a figure eight path on the floor. If we define a Cartesian x, y plane on the floor, and you wished to completely describe the bug's movements graphically, you would need to create a graph in three dimensions. This graph would need one axis to describe the x movement of the bug, another axis for its y movement, and a third axis for the time elapsed during the motion.

It is difficult to portray a graph in three dimensions on two-dimensional paper. It is relatively simple, however, to create a two-dimensional graph.

In this activity, you will model the bug's movement by breaking the motion down into its x and y components. You can then use parametric equations to separately describe each of these components as a function of time. Finally, you will use the parametric mode in your calculator to combine these graphs and create a model that describes the motion of the bug.
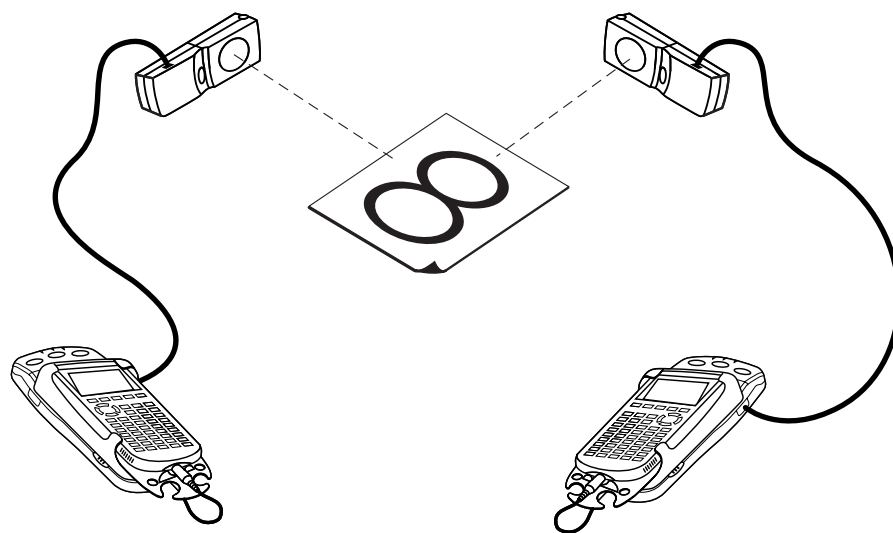
OBJECTIVES

- Record the x- and y-coordinates of a rod moving in a figure-eight pattern.
- Use the recorded motion information to separately model the x- and y- motion as a function of time.
- Plot the experimental data in a y versus x graph.
- Plot the x- and y-models parametrically for comparison to the experimental data.
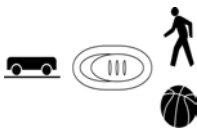
MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator(s)*
EasyData application
two CBL 2 and two Motion Detectors
    or
two CBR 2 or Go! Motion with direct calculator cables
    or
one Vernier LabPro and two Motion Detectors
dowel rod or plastic pipe, about 50 cm by 1 cm
meter stick
masking tape

*Two calculators are needed if using CBR 2, Go! Motion, or CBL 2. One calculator is needed if using a Vernier LabPro.

PROCEDURE

This activity can be done with a variety of combinations of motion detectors and data-collection interfaces. Determine the combination you will be using and carefully read the instructions for your combination.

1. Set up the Motion Detector and calculator.

    a. Turn on your graphing calculator(s).
    b. Open the pivoting head of the Motion Detectors. If your Motion Detectors have sensitivity switches, set them to Normal as shown.
    c. Connect the Motion Detectors to the calculators(s). (This may require the use of a data-collection interface.)
        - If you are using two CBL 2 data-collection interfaces, connect the Motion Detectors to the DIG/SONIC port of each.
        - If you are using two CBR 2 or Go! Motion with direct calculator cables, connect the Motion Detector directly to the calculator.
        - If you are using a LabPro, connect the Motion Detectors to the DIG/SONIC 1 and DIG/SONIC 2 ports (use Port 1 for the *x*-detector).

2. Use the figure-eight sheet from Page 6 of this activity. Tape the sheet to the table to keep the sheet from moving.

3. Set up the Motion Detectors on a table or desk as shown in the figure. Use a meter stick to place each detector 50 cm from the points indicated on the figure-eight sheet. Make note of which detector will collect *x*-data and which will collect *y*-data. Each detector must have an unobstructed view of the sheet.

4. Set up EasyData for data collection. If you are using two calculators, do this on both.

    a. Start the EasyData application, if it is not already running.
    b. Select ⌐File⌐ from the Main screen, and then select **New** to reset the application.
    c. Select ⌐Setup⌐ from the Main screen, then select **Time Graph…**
    d. Select ⌐Edit⌐ on the Time Graph Settings screen.
    e. Enter **0.2** as the time between samples in seconds and select ⌐Next⌐.
    f. Enter **50** as the number of samples and select ⌐Next⌐.
    g. Select ⌐OK⌐ to return to the Main screen.

5. Hold the rod vertically with the tip resting at the center of the figure-eight pattern. Keep your hands and arms out of the beams of the Motion Detectors. Practice moving the rod along the pattern, keeping the rod vertical the entire time. Trace the figure several times in ten seconds.

6. Select ⟨Start⟩ (on both calculators simultaneously if you are using two) to begin data collection. Then, move the rod as you practiced. You will have ten seconds to complete the motion two or more times.

7. When data collection is complete, a graph of distance versus time will be displayed on the calculator screen(s). Examine the distance versus time graphs. The graphs should show smooth sinusoidal functions with no large spikes. Check with your teacher if you are not sure whether you need to repeat the data collection.

   To repeat data collection, select ⟨Main⟩ and repeat Step 6.

8. Once you are satisfied with the graph, select ⟨Main⟩ to return to the Main screen. Exit EasyData by selecting ⟨Quit⟩ from the Main screen and then selecting ⟨OK⟩.

ANALYSIS

Since there are a lot of steps to this analysis, here is an overview of what you are about to do:

    a. Transfer the *y* data to the *x* calculator if you used two calculators.
    b. Graph the *x* versus time data, and fit a sine curve to the raw data.
    c. Graph the *y* versus time data, and fit a sine curve to the raw data.
    d. Graph the data of the *y*-detector versus the data of the *x*-detector, which will result in a picture of the figure-eight path.
    e. Superimpose the two sine models on the *y* versus *x* graph, using the parametric plotting mode of the calculator.
    f. Evaluate the quality of the parametric sine model for your motion data.

1. (Skip this step if you are using a single LabPro with one calculator.)

   To complete the analysis, you need to move the distance data from one calculator to another. In order to avoid overwriting data, first copy the data to a new location in the calculator, and then send it to the receiving calculator. Use the *y*-data calculator as the sending calculator, and the *x*-data unit as the receiving calculator. Be sure you know which is which! Once you complete the transfer, use the receiving calculator for the remainder of this activity.

       a. Disconnect the calculators from the interfaces or Motion Detectors, and directly connect the two calculators with a unit-to-unit cable. Firmly press in the cable ends.
       b. On the sending calculator, copy your distance data to a new list name. To do this, press ⟨2nd⟩ [L6] ⟨STO▸⟩ ⟨2nd⟩ [L2].
       c. You will now have the expression L6 → L2 on the home screen. Press ⟨ENTER⟩ to copy the distance data in L6 to L2.
       d. Press ⟨2nd⟩ [LINK].
       e. Press the number adjacent to List... to select a data list.
       f. Press ⟨▽⟩ until L2 is highlighted. Press ⟨ENTER⟩ to select it.
       g. Press ⟨▷⟩ to highlight the TRANSMIT menu.
       h. On the receiving calculator, press ⟨2nd⟩ [LINK].

i. Press ⊙ to highlight RECEIVE, and press ⟨ENTER⟩. The receiving calculator will show Waiting....

j. On the sending calculator, press ⟨ENTER⟩ to actually transmit the list.

k. The receiving calculator may display a message screen headed DuplicateName. If it does, press the number adjacent to Overwrite. The receiving calculator will show Done.

2. Display just the *x*-component of the rod's position versus time, after which you will fit a sine curve to the data.

   a. Press ⟨2nd⟩ [STAT PLOT] and press ⟨ENTER⟩ to select Plot 1.

   b. Change the Plot1 settings to match the screen shown here. Press ⟨ENTER⟩ to select any of the settings you change.
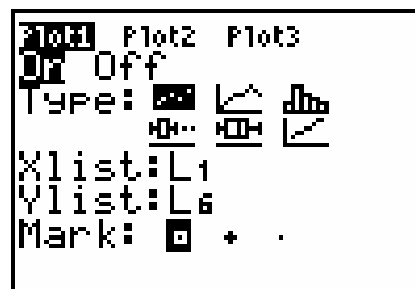
   c. Press ⟨ZOOM⟩ and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the *x* and *y* ranges set to fill the screen with data.

3. You have already seen a graph of this data, but now you are ready to fit a sine function to the data.

   a. Press ⟨STAT⟩ and use the cursor keys to highlight CALC.

   b. Press ⊙ to highlight SinReg; press ⟨ENTER⟩ to copy the command to the home screen.

   c. Press ⟨2nd⟩ [L1] ⟨,⟩ ⟨2nd⟩ [L6] ⟨,⟩ to enter the lists containing the data.

   d. Press ⟨VARS⟩ and use the cursor keys to highlight Y-VARS.

   e. Select Function by pressing ⟨ENTER⟩.

   f. Press ⟨ENTER⟩ to copy Y1 to the expression.

   g. On the home screen, you will now see the entry SinReg L1, L6, Y1. This command will perform a sine regression with L1 as the *x* and L6 as the *y* values. The resulting regression function will be stored in equation variable Y1.

      Press ⟨ENTER⟩ to perform the regression. Use the parameters a, b, c, and d that appear on the calculator screen to write down the fitted equation for the *x*. Round the fitted parameters to three significant digits and write them in the Data Table on the *Data Collection and Analysis* sheet.

   h. Press ⟨GRAPH⟩ to see the graph.

4. Using the same method you did for the *x* data, graph and fit a sine function to the *y* data.

   • If you originally collected the data using two calculators, the *y* data is in list L2; substitute that for any occurrence of L6 in the instructions above.

   • If you used a LabPro® and two Motion Detectors, the *y* data is in list L9; substitute that for any occurrence of L6 in the instructions above. (To enter L9, press ⟨2nd⟩ [LIST], scroll to L9, and press ⟨ENTER⟩ to paste it.)

   Record the resulting sine model with its four parameters as the *y* model in the Data Table.

5. So far you have fit sine curves to both the *x* and *y* data, which gives you functions for *x* and *y* as a function of time. Now you are ready to plot the position data in a *y* versus *x* graph, which will give you a graph that looks like the actual path of the rod. You can then use a parametric

graph to superimpose the separate sine curves (both functions of time) on the *y* versus *x* graph.

Display a graph of the *y* values versus the *x* values. Use the receiving calculator if you collected data using two calculators.

a. Press ⟨2nd⟩ [STAT PLOT] and press ⟨ENTER⟩ to select Plot 1.

b. Change the Plot1 settings to match the screen shown here. Press ⟨ENTER⟩ to select any of the settings you change.

(If you are using LabPro® with one calculator, use L9 for the Ylist instead of L2.

c. Press ⟨ZOOM⟩ and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the *x* and *y* ranges set to fill the screen with data.
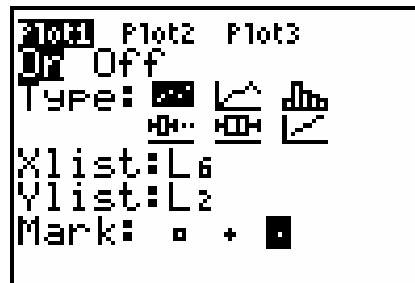
The graph you get is nearly, but not quite, what you want since the proportions of the *x* and *y* scales are not the same.

d. Press ⟨ZOOM⟩ and select ZSquare to replot the data with equal-size *x* and *y* pixels.

6. To create a parametric plot, you must change the mode of the calculator.

a. Press ⟨MODE⟩.

b. Press ⟨▽⟩⟨▽⟩⟨▽⟩⟨▷⟩ to highlight Par, and press ⟨ENTER⟩ to set the parametric graphing mode.

7. Use the following steps to enter the two sine regression equations that you earlier recorded in the Data Table. Since the parametric graphing mode needs *x* and *y* as a function of *t*, or time, substitute *t* for *x* in the expressions.

a. Press ⟨Y=⟩.

b. Press ⟨CLEAR⟩ to remove any existing equations.

c. Enter the *x* model expression in the X1T field. Press ⟨ENTER⟩ to move to the next field.
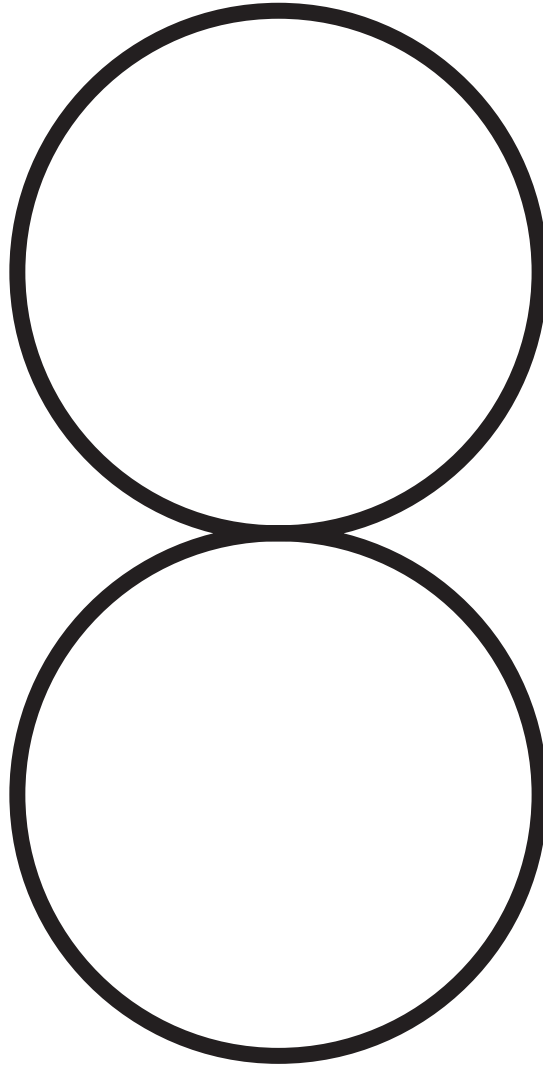
d. Enter the *y* model expression in the Y1T field.

e. Press ⟨WINDOW⟩ to adjust the plotting window.

f. Since data collection ran for ten seconds, set Tmin to zero, press ⟨▽⟩ and change the tMax setting to 10.

g. Press ⟨GRAPH⟩ to see the graph of *y* versus *x* data with the model superimposed.

⟹ Answer Questions 1 and 2 on the *Data Collection and Analysis* sheet.

Place one motion detector (to collect *x* data)
about 50 cm from this point

•

Place one motion detector (to collect *y* data)
about 50 cm from this point

•

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLE

x model equation	
y model equation	

QUESTIONS

1. How does the parametric model fit the actual *x* and *y* data?

2. In a parametric plot, how is time represented?