USE OF CAS IN TEACHING DIFFERENTIAL CALCULUS

by FIRST PRINCIPLES

We will first define the function to be $f(x) = x^2 - 4x + 3$



Consider the points on the function y = f(x) at x and x + hThe co-ordinates of these points are given above. If we use our knowledge of the gradient between points we know that the gradient of the secant will be:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

So for the points on the graph above we have:

$$m = \frac{f(x+h) - f(x)}{x+h-x}$$
$$= \frac{f(x+h) - f(x)}{h}$$





For a given function we can easily calculate this using CAS:

F1+ F2+ F3+ F4+ F5 ToolsA13ebra(ca)cBtherPr3mlD(Clean UP	F1+ F2+ F3+ F4+ F5 ToolsA19ebraCalclOtherPr9mIOClean UP
Done	■ f(x + h) - f(x)
• f(x + h)	2 · h · x + h ² - 4 · h
(×+h−3)·(×+h−1)	$2 \cdot h \cdot x + h^2 = 4 \cdot h$
■ expand((x + h = 3) ·(x + h = 1))	■ <u>2:n:x+n = + n</u> h
$\underline{x^2 + 2 \cdot h \cdot x - 4 \cdot x + h^2 - 4 \cdot h}$	2·×+h-4
expand((x+h-3)*(x+h-1)) MAIN BAD AUTO FUNC 4/30	ans(1)/h Main Bablauto Func 6/30

Hence the **gradient of the secant** between the two given points is given by 2x + h - 4. We can then discuss the idea of the horizontal distance between points becoming smaller and smaller, is as $h \rightarrow 0$, and hence the gradient of the tangent. Initially this can be calculated by evaluating the expression when h = 0.



Note the use of the $|\mathbf{h} = \mathbf{0}$ command

Consider the function	$f(x) = 2x^2$
Gradient of the secant	$=\frac{f(x+h)-f(x)}{h}$
	$=\frac{2(x+h)^2-2x^2}{h}$
	$=\frac{4hx+2h^2}{4hx+2h^2}$

$$\frac{1}{h} = 4x + 2h$$

Gradient of the tangent = $\lim_{h \to 0} (4x + 2h)$ =4x

F1+ F2+ Tools Algebr	aCalcOther P	F5 r9m10(F6+ :1ean Up	
expand	(f(x + h) ·	- f(×))))	-
		4∙h∙	x + 2 ·	h ²
■ 4 · h · × · h	+2·h ²	2.1	(2·×+	۰h)
■2·(2·×	+ h) h = (9		4·×
ans(1)	n=0			
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F1+ F2+ Tools Algebr	aCalcOther	F5 Pr9mi0	F6+ Clean Up	\square
∎ <u>f(x+h</u>	h - f(x)	2	(2·×·	+ h)
expand	 (2·(2·×·	+ h))		
			4 · × + ∶	2∙h
■ lim(4	·×+2·h)	1		4∙×
limit(4)	*x+2*h,ł	1,0)		
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Use the following functions to find the gradient of the tangent to the curve at any point on the curve:

1.
$$f(x) = x^2$$
 2. $f(x) = x^3 - 2x + 1$ 3. $f(x) = x^4 - 2x^3 + 1$

Ensure that all working is shown. Include in your calculations the expressions for f(x+h), f(x+h) - f(x) and $\frac{f(x+h) - f(x)}{h}$

How would your calculations change if you want to evaluate the gradient at a given point?

F1+ F2+ F3+ F4+ F5 ToolsAl9ebraCalcOtherPr9r	F6+ nIOClean UP	
	2·×+h−4	
■ 2·×+h-4 h=0	2·× - 4	
$\frac{f(1+h)-f(1)}{h}$	h – 2	
■h-2 h=0	-2	
ans(1) h=0 MAIN RADAUTO	FUNC 9/30	Here we can see that we can use the point $x = 1$.

NOTE(for teachers): It can be seen that the use of CAS eliminates the occurrence of incorrect evaluation and simplification of expressions. Also, it is possible to go through this procedure many times in a short period of time.