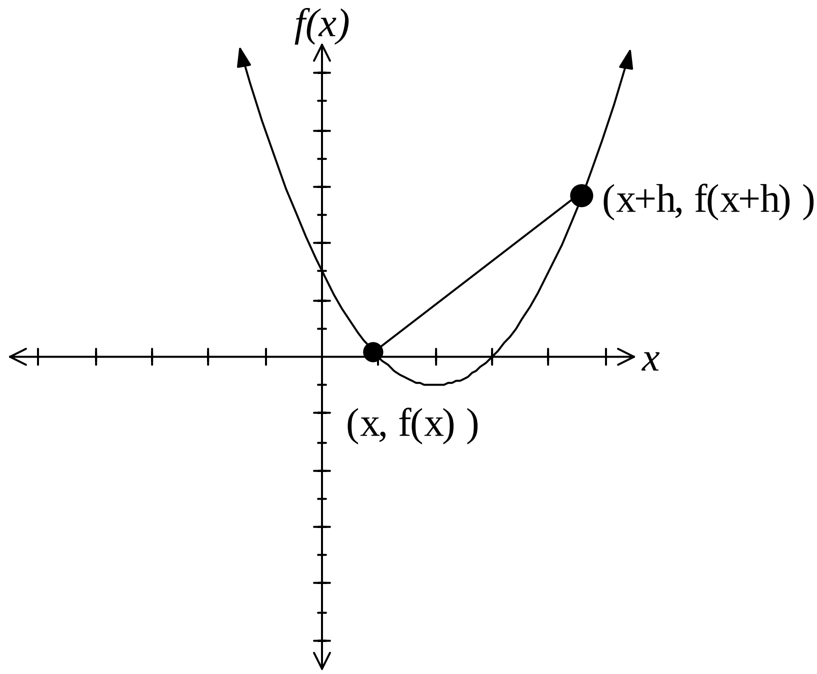
USE OF CAS IN TEACHING DIFFERENTIAL CALCULUS

by FIRST PRINCIPLES

We will first define the function to be $f(x) = x^2 - 4x + 3$



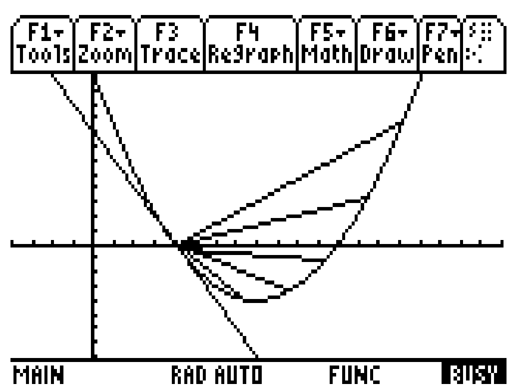
Consider the points on the function $y = f(x)$ at x and $x+h$
The co-ordinates of these points are given above. If we use our knowledge of the gradient between points we know that the gradient of the secant will be:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

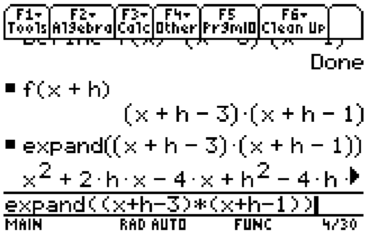
So for the points on the graph above we have:

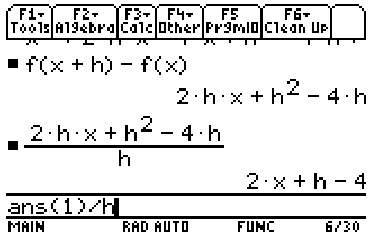
$$m = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

NOTE: It is possible to animate a sequence of pictures, which will demonstrate the idea of a secant approaching the tangent at a point.

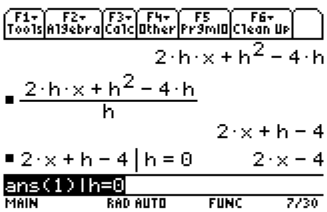


For a given function we can easily calculate this using CAS:



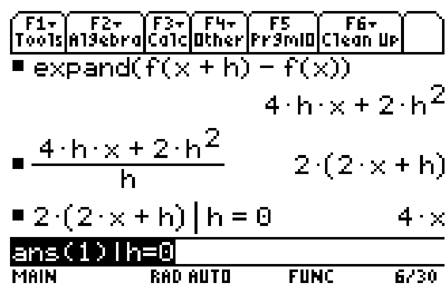


Hence the **gradient of the secant** between the two given points is given by $2x + h - 4$. We can then discuss the idea of the horizontal distance between points becoming smaller and smaller, ie as $h \to 0$, and hence the **gradient of the tangent**. Initially this can be calculated by evaluating the expression when h =0.

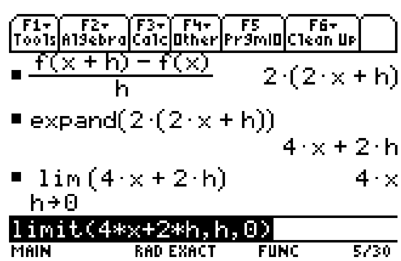


Note the use of the **| h = 0** command

Consider the function $f(x) = 2x^2$



$$
\begin{aligned}
\text{Gradient of the secant} &= \frac{f(x+h) - f(x)}{h} \\
&= \frac{2(x+h)^2 - 2x^2}{h} \\
&= \frac{4hx + 2h^2}{h} \\
&= 4x + 2h
\end{aligned}
$$



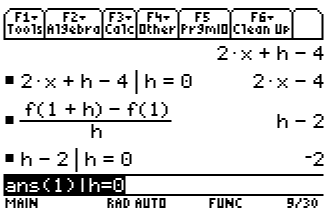
$$
\begin{aligned}
\text{Gradient of the tangent} &= \lim_{h \to 0}(4x + 2h) \\
&= 4x
\end{aligned}
$$

Use the following functions to find the gradient of the tangent to the curve at any point on the curve:

1. $f(x) = x^2$ 2. $f(x) = x^3 - 2x + 1$ 3. $f(x) = x^4 - 2x^3 + 1$

Ensure that all working is shown. Include in your calculations the expressions for $f(x+h)$, $f(x+h) - f(x)$ and $\frac{f(x+h) - f(x)}{h}$

How would your calculations change if you want to evaluate the gradient at a given point?



Here we can see that we can use the point $x = 1$.

NOTE(for teachers): It can be seen that the use of CAS eliminates the occurrence of incorrect evaluation and simplification of expressions. Also, it is possible to go through this procedure many times in a short period of time.