Exploring Quadratic Equations

Problem Statement

There are times when we use a graph to solve a problem and there are times when using a graph is difficult. Suppose the functions used in a problem are such that you have no idea how to set your viewing window in order to see key features of the graph needed for a solution. During these times, an algebraic approach might be easier. In order to be able to use both approaches, we need to understand how the graphical solutions are related to the algebraic solutions. In this investigation, we will build these connections.

Stretching a Parabola

1. On page 1.4 of the *CollegeAlg_ExpQuadratics.tns* file is a graph of the function $f(x) = x^2$. Move your cursor to a branch of the parabola and you should see the cursor change to ⤡. Press and hold the Click key to grab the parabola. To stretch the parabola, use the NavPad to move the cursor and click again to place the graph.

a. Describe how the equation changes as you manipulate the graph.

b. When the coefficient of x^2 becomes negative, what happens to the graph?

c. On page 1.7, is the coefficient of x^2 positive or negative? Explain your reasoning.

d. What is a possible coefficient of x^2 in the graph on page 1.8 of the *CollegeAlg_ExpQuadratics.tns* file?

Translating a Parabola

2. On page 2.2 is a graph of the function $f(x) = x^2$. Move the cursor to the origin until you see the cursor change to ✣. Grab the parabola and move it around the coordinate plane.

a. Describe how the equation changes as you manipulate the graph.

b. Recall that the vertex-form equation for a quadratic is given by $y = a(x - h)^2 + k$. By examining the values for h and k in the algebraic expressions for various graphs, make a conjecture for how the point (h, k) is related to the features of the graph.

c. Based on the algebraic expression for the parabola, what is the vertex of the graph on page 2.5 of the *CollegeAlg_ExpQuadratics.tns* file? Does it match your expectations from the graph?

d. Based on the algebraic expression for the parabola, what is the vertex of the graph of the function $f(x) = (x - 3)^2 + 1$?

e. Which of the following functions has (have) a vertex at (–1, 1)?

$a(x) = 2(x - 1)^2 + 1$

$b(x) = -1(x + 1)^2 - 1$

$c(x) = -3(x + 1)^2 + 1$

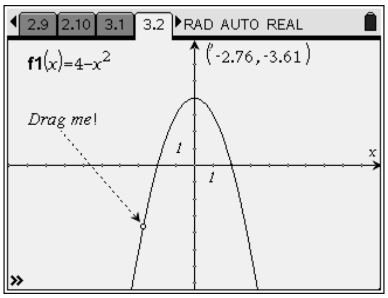
f. Write an equation with a vertex of (–2, 3). Check your work by graphing it on page 2.9 of the *CollegeAlg_ExpQuadratics.tns* file.

g. Write a second equation with a vertex of (–2, 3), if possible. If it is not possible, explain why.

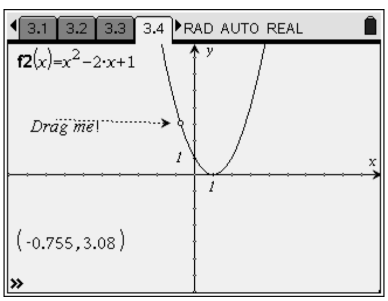
Finding Zeros of a Quadratic Graphically

3. For the graphs on pages 3.2, 3.4, and 3.6 of the *CollegeAlg_ExpQuadratics.tns* file, grab the point on the graph and move it to find the maximum/minimum and zeros (these words will appear when you reach each point).

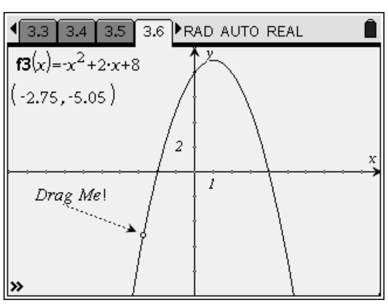
a. What is (are) the zero(s) of the function on page 3.2? Label your zeros on the graph below.



b. What is (are) the zero(s) of the function on page 3.4? Label your zeros on the graph below.



c. What is (are) the zero(s) of the function on page 3.6? Label your zeros on the graph below.



Connecting Zeros to the Equation

4. In this part of the investigation, we will find the zeros for each given function. To do this graphically, on each of pages 4.3, 4.4, and 4.5, select **MENU > Points & Lines > Intersection Points** to find the intersection between the parabola and *x*-axis to determine the zeros. Select the graph and then the *x*-axis.

a. Describe how the factored forms of the function at the bottom of each page can help find the zeros. For the factored-form equation, $y = a(x - p)(x - q)$, what do p and q represent?

b. What are the zeros of the function given on page 4.8? What general form must the algebraic expression of the function take?

c. In looking at the zeros you found for the parabolas on pages 4.3, 4.4, and 4.5, compare the values of the zeros for each with the vertex of the parabola. Describe any patterns you see between the zeros and *x*-coordinates of the vertices.