Teachers Explanatory Notes

TI-15 Explorer™: Function Machines & Factor That!

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This unit aims at improving students' understanding of various operations (+, –, x and ÷); in particular they will explore the relationship between operations and inverse operations. After exploring various operations using function machines the students will explore prime and composite numbers as a prelude to finding prime factors using factors trees. More able students can further explore factors using the TI-15 Explorer™.

1. Students find the *problem-solver* feature of the TI-15 a lot of fun, as the calculator asks them the questions. Allow students to use this feature to try some questions. This will act as a "warm up" for this series of lessons and may be used at the start of any of the lessons.

   If the students haven't used this feature explain how it works. (There is also a short video tutorial on how to use this feature in the video tutorial section of this CD).

Press the problem solving key ◈

The calculator will ask the students to solve questions. Initially the calculator is set to ask simple single-digit addition questions.

This initial setting may be altered by pressing the mode key (Mode) once the problem solving key ◈ has been pressed once. YES NO will briefly appear then fade from the screen.

After a second or two the display will show <u>AUTO</u> MAN, the selected setting is always the underlined one.

Use the left/right arrows to change your selection. Use the up/down arrows to scroll through the various options.

1, 2, or 3 indicates the level of complexity of the question, with 1 being the simplest and 3 the hardest. The level may be altered by scrolling to the right so that either 2 or 3 is underlined. Pressing enter then executes this choice. The questions will become harder.

Of more interest is the ability to change the operation that is used. If the operation was initially set at addition all that a level change will do is make the addition questions harder, that is, two or three-digit additions. To change the operation that is used the student will need to press the mode key, wait for the YES NO recording to fade from the screen. The display will show 1, 2, or 3 and one of these levels will be underlined indicating the set level. Scrolling down once, will take you to a display that shows + – x ÷ and ? Underlining the subtraction, multiplication or division symbol changes the operation from addition to the one that is underlined. To underline a different operation you scroll across and when the line is under the appropriate operation you press enter.

After allowing sufficient time for the students to try questions involving the four operations (+ – , x and ÷) at different levels then introduce the ? option. Rather than provide a question to be solved in the standard format of 3 x 4 = ?, it is provided in the format 3 ? 4 = 12 and the student has to work out what operation is being used.
As students gain experience with this feature discussion can take place around the idea of inverse operations and the notion that one operation may be used to undo another. This is particularly useful for linking addition & subtract and multiplication & division. These links will be built on when working with Function Machines (See 2 below).

2. **Function Machines (Single Forward)** may be used to help students explore and formalise the idea of combining operations and inverse operations. In the first instance numbers are fed into the function machine and come out the other end. Students have to work out what is happening to the numbers inside the function machine.

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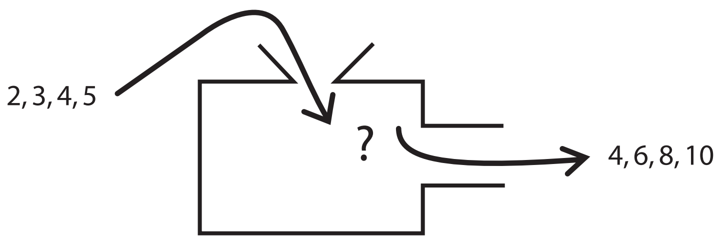


For example, if the numbers 2, 3, 4 and 5 are inserted into the function machine and the numbers 4, 6, 8 and 10 come out the other end then it would suggest that the machine is doubling the numbers. Alternatively the students could write '× 2'. Several different examples could be provided where the students have to work out what is happening to the numbers that are inserted into the machine.

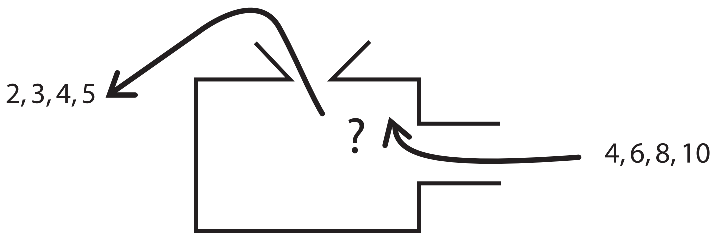
Once the students understand the idea of the function machine the [Op1] key can be introduced. Explain that the Operation keys ([Op1] and [Op2]) can act like function machines. For example pressing the key sequence [Op1][+][5][Op1] sets the TI-15 Explorer™ calculator to add five every time the [Op1] key is pressed. For example, if the [Op1] key is set to add five then pressing [1] [Op1] will produce 6 on the display, pressing [2] [Op1] produces 7, [3] [Op1] produces 8 and so on. Some students will notice that this is similar to the way function machines work.

W1

Hand out copies of Worksheet 1 "**Single Function Machines**". Set students the task of creating a series of function machines for other students to solve. Remind the students to keep a copy with the answers and then create a copy for a partner to solve that only has the entry and exit number showing. The students can make use of the [Op1] key on the TI Explorer™ calculator to assist them to create the problems for their partner to solve.

3. Reverse Function Machines

The idea of an inverse operation may be formalized by returning to the idea of function machines. Ask the students to consider what happens if the function machine is set to reverse.



Some students may recognize that the numbers fed into the machine and the numbers that come out are similar to the first example. In this case, however, instead of the numbers being doubled, they are being halved. Students may express this idea in different ways, for example, x 0.5, x ½, or dividing by two. Discuss the relationship between these different representations.

W2

Hand out copies of Worksheet 2 "**Reverse Function Machines**". Ask students to create a set of questions where other students are provided with the entry and exit numbers but the machine is running in reverse. Students make two copies of their reverse function machines. One would contain the entry and exit numbers while the other would contain the answers, that is entry, exit numbers and the operation in the machine.

4. Joining Function Machines

The TI-15 Explorer™ calculator has two different operation keys, labeled [Op1] and [Op2]. Some interesting investigations may be developed using these two keys in unison.

Introduce the idea of joining function machines using a story shell framework. Explain that in a factory two machines are required to extrude plastic. One machine adds five cm to the length of plastic while a second machine adds 3 cm. So if a piece of plastic 10 cm long is placed in machine 1, it comes out 15 cm long. The 15 cm length, when fed into machine two comes out 18 cm long. The two machines are linked so you don't see the 15 cm long piece. A 10 cm piece goes in and an 18 cm piece comes out. Explain that one day the +5 cm machine breaks down and cannot be repaired. The engineer explains that it would be more efficient to replace the two machines with a single machine. What type of machine should you buy? Students can experiment on the TI-15 Explorer™ using the [Op1] and [Op2] keys to determine how a single machine could be used to replace the other two.

Further challenges may be set involving addition, eg + 2 + 3, + 4 + 4 etc. Students will soon discover that the two amounts may be added together to achieve the same result. Set further challenges involving two machines that subtract eg – 3 – 4, multiply x 2 x 4, and divide, ÷ 2 ÷ 5.

Caution: Some students may over-generalise once they work out that the two amounts to be subtracted may be added together, eg – 3 – 4 becomes – 7. Watch for students who suggest that x 2 x 4 may be replaced by a x 6 machine. A little experimentation with the [Op1] and [Op2] keys will soon show that x 2 and x 4 is the same as x 8. Likewise ÷ 2 and the ÷ 5 is the same as ÷ 10.

Worksheet 3 "**Joining Function Machines**" has been provided so students may explore and formalise what happens when two operations are linked. The [Op1] and [Op2] keys will assist in the development of this understanding.

W3

5. Prime Numbers

Mathematics is about the search for and discovery of patterns. The second part of this unit will explore factors.

To gain a robust understanding of factors students need to be able to identify prime and composite numbers. The following instructions will help students distinguish prime and composite numbers. A chart showing the prime numbers up to 200 has been provided in the appendices.

The Sieve of Eratosthenes activity can be done with the whole class with and interactive white board, in groups or as a home task.

Sieve of Eratosthenes

Eratosthenes (275-195 BC). Eratosthenes is often referred to as the librarian who measured the world. He was the chief librarian at the library of Alexandria. He made use of shadows and geometry to calculate the circumference of the Earth. His calculation was remarkably accurate, especially since he didn't have a TI-15 Explorer™ calculator to assist him with the calculation.

Eratosthenes developed a system for determining prime numbers. We can use a similar system for working out prime numbers. Here are the instructions for you to try.

1. Draw a circle around 2 and cross out all the multiples of two.
2. Draw a circle around 3 and cross out all the multiples of three.
3. Draw a circle around 5 and cross out all the multiples of five.
4. Draw a circle around 7 and cross out all the multiples of seven.

The numbers that remain, along with the circled numbers represent all the prime numbers from 1 – 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

See *http://www.math.utah.edu/~pa/Eratosthenes.html* for a java applet demonstrating the Sieve of Eratosthenes.

Students can now complete Worksheet 4 in class or at home. This worksheet has reconfigured the table to better show the pattern formed by prime numbers. Students will need a list of the prime numbers they have found to complete this at home. W4

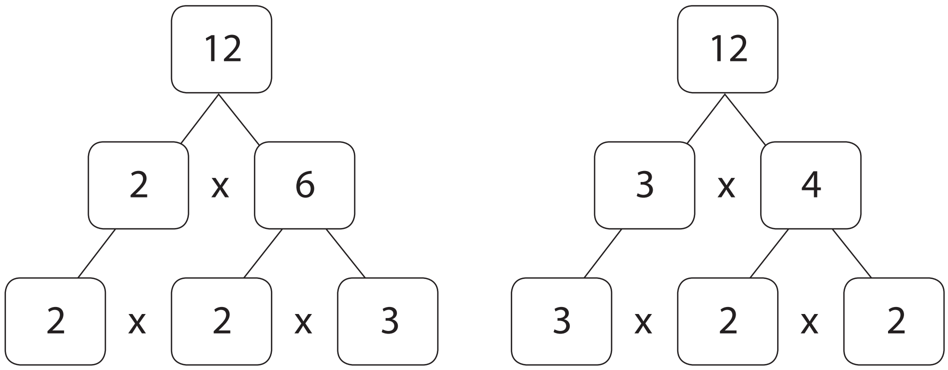
6. Extension – Factors and Factor Trees

(There is another unit dealing entirely with factors on this resource CD)

The idea of finding factors for numbers may be illustrated using factor trees. With factor trees you keep finding factors of a number (branching out – hence the name factor tree) until you reach prime numbers and can go no further. The final set of factors is called prime factors.

Factor: Any counting number that divides another number without any remainder is a *factor* of that number. For example 1, 2, 3, 6, 9 and 18 are all factors of 18.

Sometimes it is helpful to express a number in terms of prime factors. A *factor* tree may be used to work out the prime factors of numbers. A *factor tree* is a diagrammatic strategy used to factorise, that is find all the prime factors of a counting number. Here are two different factor trees for 12.



Notice how the numbers are broken down into factors until you reach prime numbers and cannot go any further.

Create your own factor trees for the following numbers. As in the example above there may be more than one way to factorise the number but eventually you will end up with the same prime factors.

1. 18
2. 36
3. 40
4. 48
5. 51
6. 60
7. 73
8. 86
9. 95
10. 99

Factor Tree: Answers

1. 18; 2 x 3 x 3
2. 36; 2 x 2 x 3 x 3
3. 40; 2 x 2 x 2 x 5
4. 48; 2 x 2 x 2 x 2 x 3
5. 51; 3 x 17
6. 60; 2 x 2 x 3 x 5
7. 73; prime number
8. 86; 2 x 43
9. 95; 5 x 19
10. 99; 3 x 3 x 11

7. Exploring Factors on with the TI-15 Explorer™

- Students can set the TI-15 Explorer™ to explore factors. This involves extending the problem solving feature of the calculator that began this sequence of lessons. Press the problem key ◈, then press mode (Mode) and scroll to the AUTO MAN screen. Choose the manual option by scrolling across and pressing enter. This will allow one student to set a question and then hand the TI-15 to another student to answer the question. (There is also a short video tutorial on how to use this feature in the video tutorial section of this CD.) In this mode, the student writing the question can use "?" as a *pronumeral* in an equation to be solved. Discuss the meaning of a *pronumeral* and the idea of evaluating a *pronumeral* to "solve" an equation – an explanation has been included in the **Appendix**.

- Ask students what happens on the calculator display when the **ENTER** key is pressed to enter an equation into the problem-solver while in manual mode.
  - *[1 SOL is briefly displayed indicating that there is only one value (solution) for the ? that will make the equation true.]*
  - Show that by using two "?" symbols in an equation, there may be more than one solution (Note that the different instances of "?" in the equation can have different values.)

- Have students use their calculators to investigate and record the number of whole number solutions to equations of the type $? \times ? = 1$, $? \times ? = 2$ $? \times ? = 3$, etc, up to $? \times ? = 36$.

- Discuss what the solutions may be and have students use their calculators to check. (e.g. $? \times ? = 12$ has 6 solutions. They are 1×12, 12×1, 2×6, 6×2, 3×4, and 4×3.

- Explain that the values that provide solutions are the *factors* of the target number.
  - (i.e. 1, 2, 3, 4, 6, 12 are factors of 12.)

- Provide students with formal definitions of:
  - Prime number
  - Composite number
  - Square number

- Ask students to identify numbers that have:
  - exactly 2 solutions/factors [Prime numbers]
  - more than 2 solutions/factors [Composite numbers]
  - an odd number of solutions/factors [Square numbers]

Appendix

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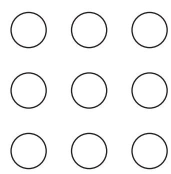
Teacher Explanations

A *pronumeral* is a symbol for a numeral in an expression or equation. For example in the number sentence $2y + 3 = 13$, y is a *pronumeral* for 5.

Prime Number: A counting number with exactly two factors; ie 2, 3, 5, 7, 11, 13 … [Note that 1 is not a prime number as it has only one factor and like zero is considered a special number]

Composite Number: Any counting number having more than two factors; i.e. 4, 6, 8, 9 …

Square number: Any counting number obtained by multiplying a number by itself; i.e. 1, 4, 9, 16 … It is called square because a square number of objects can be placed in a square array.



Prime and composite numbers to 200

1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	49	50
51	52	**53**	54	55	56	57	58	**59**	60
61	62	63	64	65	66	**67**	68	69	70
71	72	**73**	74	75	76	77	78	**79**	80
81	82	**83**	84	85	86	87	88	**89**	90
91	92	93	94	95	96	**97**	98	99	100
101	102	**103**	104	105	106	**107**	108	**109**	110
111	112	**113**	114	115	116	117	118	119	120
121	122	123	124	125	126	**127**	128	129	130
131	132	133	134	135	136	**137**	138	**139**	140
141	142	153	144	145	146	147	148	**149**	150
151	152	153	154	155	156	**157**	158	159	160
161	162	**163**	164	165	166	**167**	168	169	170
171	172	**173**	174	175	176	177	178	**179**	180
181	182	183	184	185	186	187	188	189	190
191	192	**193**	194	195	196	**197**	198	**199**	200