Math Objectives

- Students will identify the standard deviation as a measure of spread about the mean of a distribution and loosely interpret it as an "average" or typical distance from the mean.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practices).

Vocabulary

- deviation
- dotplot
- mean
- range
- residual
- standard deviation

About the Lesson

- This lesson involves gaining a basic understanding of what standard deviation is measuring by examining the location of data around the mean.
- As a result, students will:
    - Find the sum of the residuals to be 0 and consider both squaring and absolute value to eliminate the negative values.
    - Compare the standard deviation to the mean of the absolute values of the residuals as the data changes.
    - Change the amount of snowfall by moving data points and comparing the squared standard deviation and the mean of the squared residuals in the plots.

Prerequisite

Students should be familiar with the concepts of mean, range, and standard deviation.

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- Use Screen Capture to show that distributions with the same mean and range can have different standard deviations.
- Use Live Presenter to demonstrate how to find two different distributions that have the same mean and standard deviation.
- Use Quick Poll to assess students understanding of residuals or
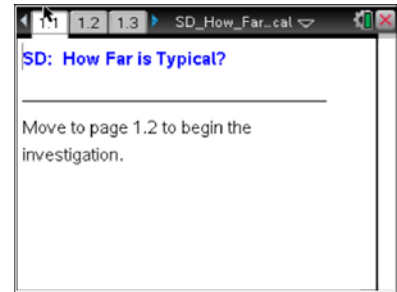
TI-Nspire™ Technology Skills:
- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab, drag and deselect a point in a dotplot

Tech Tips:
- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
SD_How_Far_is_Typical_Student.pdf
SD_How_Far_is_Typical_Student.doc
TI-Nspire document
SD_How_Far_is_Typical.tns

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standard deviation.

Discussion Points and Possible Answers

> **Tech Tip:** To de-select a point, move the cursor to a white space on the screen and click. If you do not de-select a point you have moved, it will move again along with the next point you choose.

> **Teacher Tip:** Although residual is the statistical term, the term deviation is commonly used and can help students understand the concept of standard deviation. You might want to reinforce deviations as distances from the mean. Note that standard deviation is not the mean of the residuals and also is not the mean of the absolute value of the residuals, and so this is a good time to discuss the "loose" terminology for the common interpretation of standard deviation.

Move to page 1.3.

The dotplot at the bottom of this page displays the amount of snowfall (in inches) for the six largest cities in a Midwestern state during one winter month for a given year. The vertical line segment indicates the value of the mean amount of snowfall for these cities.
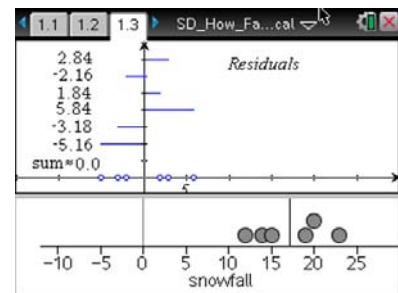
The difference between the amount of snowfall for a given city and the mean amount of snowfall for all cities (Actual for the city – Mean of all cities) is called the *residual* for that city. This difference can also be referred to as the *deviation from the mean* for that city.

> **Teacher Tip:** The use of the term residuals is often overlooked in describing univariate data.

1. If the amount of snowfall for a city is changed on the dotplot, the residuals change.
   a. How many residuals are shown? Why?

   Answer: There are six residuals; each represents the difference between the amount of snowfall for a particular city and the mean amount of snowfall in the six cities for that month.

b. What are the largest and smallest values you can get for the sum of the residuals? Explain.

    <u>**Sample Answers:**</u> The sum of the residuals is always 0 since the positive and negative residuals cancel each other out**.**

c. Is the mean of the residuals a good measure for describing the spread of the snowfall amounts? Why or why not?

    <u>**Sample Answers:**</u> No. Since the sum is always 0, the mean will always be 0.

TI-Nspire Navigator Opportunity: *Quick Poll (True or False)*
See Note 1 at the end of this lesson.

If you want to use the residuals to create a measure of spread, there are two ways to deal with negative residuals. You can take the absolute value of the residuals, or you can square the residuals. The questions that follow investigate both of these options.

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2. The top graph on page 1.4 displays the absolute values of the residuals from page 1.3, and the vertical line represents the standard deviation.
a. Suppose you could vary the amount of snowfall for the six cities, to represent different years. Move the data points to investigate the relationship between the value of the standard deviation and the absolute value of the residuals.
b. How does the value of the standard deviation seem to be related to the residuals?
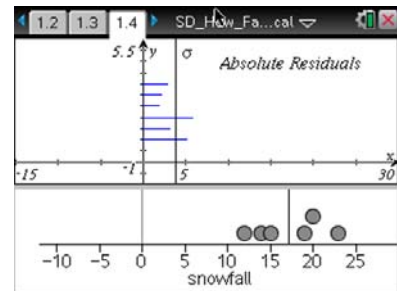
    <u>**Sample Answers:**</u> The value of the standard deviation seems to fall somewhere in the middle of the residuals.

> **Teacher Tip:** Recall that in question 1, students found that the sum and average of the residuals were always 0. Students might need to be reminded of this.
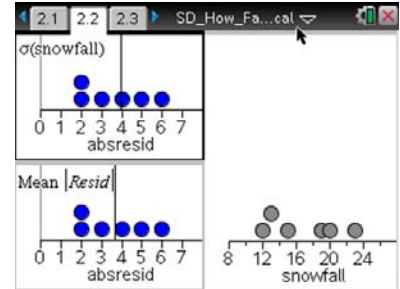
> **Teacher Tip:** The difference in using the absolute values of the residuals
> instead of using the squares of the residuals is that the mean minimizes
> the sum of squares of the residuals in a data set, and the median
> minimizes the sum of the absolute values of the residuals. Since the
> standard deviation is based on the mean, the calculation involves the
> squared residuals in order to be consistent.

Move to page 2.2

3. The amount of snowfall (in inches) during one
    winter month for the same six cities is displayed in
    the dotplots on the right-hand side of the screen.
    *   The upper left plot displays the standard
        deviation of the snowfall amounts as a vertical
        line.

    *   The vertical line in the lower left plot displays the mean of the absolute values of
        the residuals obtained from the differences between the amount of snowfall (in
        inches) for a particular city and the mean amount of snowfall for all of the cities.
    a. Change the amounts of snowfall by moving the data points in the right plot, and
        observe the changes in the plots on the left.
    b. How does the mean of the absolute values of the residuals compare to the
   standard deviation?

    Sample Answers: They are not equal, and it seems that the mean of the absolute
    values of the residuals is smaller.

    > **Teacher Tip:** The mean of the absolute values of the residuals is called
    > the mean absolute deviation (MAD).

Move to page 2.3

4. Suppose you squared the residuals to eliminate negative
    residuals instead of using the absolute values of the residuals.
    *   The amount of snowfall (in inches) for one month for the
        same six cities is displayed in the dotplot on the right hand
        side of the screen.
    *   On the left hand side of the screen, the squared residuals
        are plotted in both dotplots.
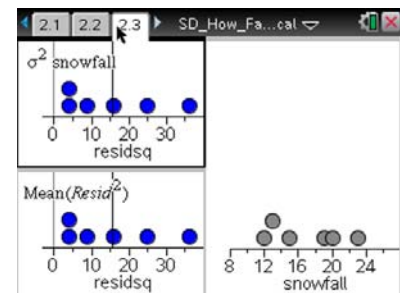
a.   Recall that a residual is the difference between the amount of snowfall (in inches) in a particular
     city during one winter month and the mean amount of snowfall for all of the cities in that month.
     What are the units of the squared residuals? Explain.

     Answer: Since residuals are [amount of snowfall (in inches) – mean amount of snowfall (in
     inches)], then the unit for a residual is inches. Therefore, the squared residuals are in (inches)^2.

- The upper left plot displays the square of the standard deviation.
- The lower left plot displays the mean of the squared residuals.

b.   Change the amount of snowfall by moving the data points in the right plot, and observe the
     squared standard deviation and mean of the squared residuals in the plots on the left.

c.   How does the value of the standard deviation squared compare to the value of the
     mean of the squared residuals?

     Answer: The values of the squared standard deviation and mean of the squared
     residuals seem to be equal.

d.   In what units is the squared standard deviation measured? Why might this be
     problematic, and what suggestions do you have to address the problem?

     Answer: (inches)^2; This is problematic because the original units are inches. You
     might take the square root to get rid of the inches squared;

> **Teacher Tip:** The square of the standard deviation is the variance.
> Question 4c explains why you take the square root of the variance to find a
> measure of a typical deviation from the mean (i.e. the standard deviation).
> This might be a good place to introduce variance to students.

5.   a.   Based on your answer to question 4, write a description of what standard deviation measures.

          Sample answer: In question 3a, the standard deviation did not appear to be
          related to the absolute value of the deviations. But since (standard deviation)^2 is
          equal to the mean of the (residuals)^2 and residuals are distances from the mean,
          the standard deviation is a typical or "average" distance from the mean.

b. Suppose the large cities in two different states have the same mean amount of rain per year, but the standard deviation for cities in one state is substantially larger than that for cities in the other state. What does that tell you about the amount of rain in the two states?

Sample answer: The amounts of rain in the largest cities in one state are relatively the same, where in the second state, with the largest standard deviation, the amounts of rain varies more from city to city.

> **Teacher Tip:** You might have a discussion about possible reasons for the differences in the two standard deviations. For example, the state with little variation might be a state that has mostly the same terrain while the state with the large amount of variation (bigger standard deviation) might be on the coast but also have mountains.

Wrap Up

Upon completion of the activity and discussion, the teacher should ensure that students are able to understand:

- Standard deviation as a measure of spread of a distribution around the center, which can be loosely interpreted as an "average" or typical distance from the mean.

Assessment

1. Suppose a researcher examines the number of texts sent per month by teenagers. The report states the mean number of texts sent per month was 1500 and the standard deviation was 315 texts per month. Interpret the standard deviation in the context of this problem.

Sample answer: The number of texts sent per month by a typical teenager differs from 1500 by around 315 texts per month.

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Note 1

Question 1c, *Quick Poll (True or False)*

a. The sum and the average of the residuals is always 0. (T)

b. The standard deviation is usually measured in terms of distance from the median of the data. (F)

c. If the standard deviation is 100, the distribution has a very large spread. (F)

> **Teacher Note**: Determining whether a standard deviation is large or small depends on the context. For example, the standard deviation for the mean income of a worker in the United States would have a standard deviation much larger than 100, while a standard deviation of 100 would be very large for a set of values representing the weight of a group of teen-agers.

d. You can calculate the standard deviation by using either the square of the residuals or the absolute value of the residuals. (F)

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