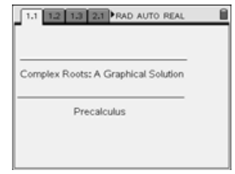
Complex Roots: A Graphical Solution

Teacher Notes

Introduction

In this activity, you will explore the relationship between the complex roots of a quadratic equation and the related parabola's graph. Open the file Complexroots_TCH.tns on your TI-Nspire™ handheld device.



Supplies/Materials

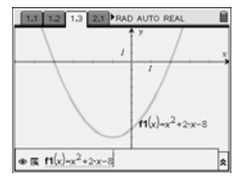
TI-Nspire CAS handheld devices

Instructions

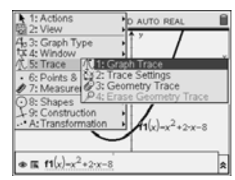
Advance to Page 1.2 and recall that the real solutions/roots/zeros of a quadratic equation of the form $ax^2 + bx + c = 0$ are the x-intercepts of its related parabola's graph and can be represented by one of two situations.

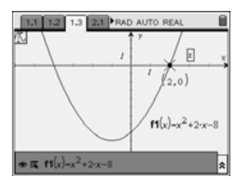
Situation 1

1. Advance to Page 1.3 by pressing (ctrl) and the right side of the NavPad.



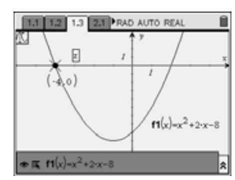
2. Examine the graph of the function $f1(x) = x^2 + 2x - 8$ and locate the exact solutions/roots/zeros.
   - Select (menu), choose 5:Trace, 1: Graph Trace, and cursor using the NavPad until zero displays.
   - Continue to cursor using the NavPad or type a likely value and then press (click) to locate the other zero when zero displays.



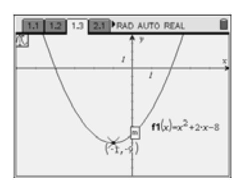


Q1: What are the real solutions/roots/zeros of $x^2 + 2x - 8 = 0$?

$x = -4$ and $x = 2$



3. Locate the vertex of this parabola.
   - Select (menu), choose 5:Trace, 1: Graph Trace, and cursor using the NavPad until minimum displays.



Q2: Name the axis of symmetry and the coordinates of the vertex of the graph of $f1(x) = x^2 + 2x - 8$.

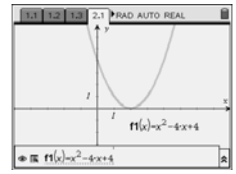
AS: $x = -1$; V: $(-1, 9)$

Q3: Describe the location of the real zeros with respect to the axis of symmetry and the vertex.

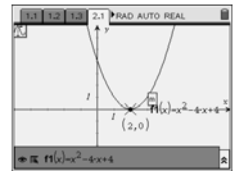
The line segment joining the real zeros is perpendicular to the axis of symmetry. The real zeros are equidistant from the axis of symmetry and the vertex.

Situation 2

4. Advance to Page 2.1 by pressing (ctrl) and the right side of the NavPad.
5. Examine the graph of the function $f1(x) = x^2 - 4x + 4$ and locate the exact solutions/roots/zeros and the vertex.
   - Select (menu), choose 5:Trace, 1: Graph Trace, and cursor using the NavPad.



Q4: How many distinct real solutions/roots/zeros exist?
1



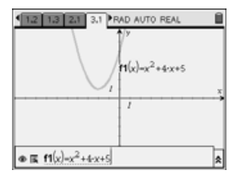
Q5: What are the distinct real solutions/roots/zeros of $x^2 - 4x + 4 = 0$?
x = 2

Q6: Name the axis of symmetry and the coordinates of the vertex of the graph of $f1(x) = x^2 - 4x + 4$.
AS: x = 2; V: (2, 0)

Q7: Describe the location of the real zeros with respect to the axis of symmetry and the vertex.
The real zero lies on the axis of symmetry and passes through the vertex.

Finding Complex Roots

6. Advance to Page 3.1 by pressing (ctrl) and the right side of the NavPad and view the graph.

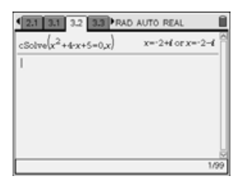


Q8: Name the axis of symmetry and the coordinates of the vertex of the graph of $f1(x) = x^2 + 4x + 5$.
AS: x = -2; V: (-2, 1)

Q9: What are the real solutions/roots/zeros?
There are no real solutions/roots/zeros.

Q10: How can you tell from the graph of a parabola whether real or complex zeros exist?
Real zeros exist if the parabola intersects the x-axis. Complex zeros exist if the parabola does not intersect the x-axis.

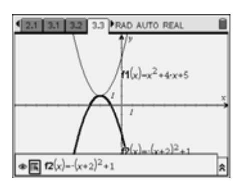
7. Advance to Page 3.2, find the complex solutions of $x^2 + 4x + 5 = 0$, and express in $a + bi$ form.
   - Select (menu), choose 3: Algebra, A: Complex, and 1: Solve.
   - Type $x^2 + 4x + 5 = 0$, x inside the parentheses as shown and press (enter).



Q11: What are the complex solutions of $x^2 + 4x + 5 = 0$?
-2 + i and -2 - i

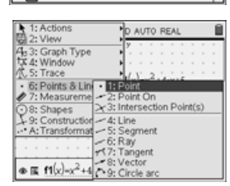
Visualizing Complex Roots

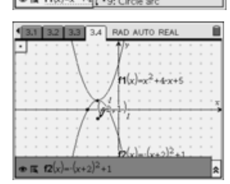
8. Advance to Page 3.3, use the *k* value in the vertex form of the graph of the function $f1(x) = x^2 + 4x + 5$, and reflect the parabola over $y = k$.
   - Enter the reflected function for *f2(x)* and graph.



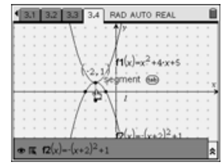
Complex numbers of the form $a + bi$ are graphed by using the *x*-axis as the real axis for *a* and the *y*-axis as the imaginary axis for *bi.*

9. Advance to Page 3.4 and plot the complex roots.
   - Select (menu) and choose 6: Points & Lines and 1: Point.
   - Move pencil to each complex root (*point on* will display) and press (enter) or (click).
   - Press (esc) to exit this menu.

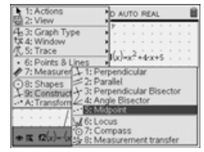


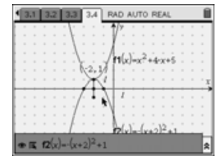


10. Draw the segment joining the plotted complex roots.
    - Select menu and choose 6: Points & Lines and 5: Segment.
    - Cursor to each plotted complex root and press enter or click and esc to exit this menu.

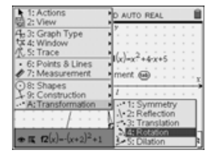


11. Locate the midpoint of the segment joining the plotted complex roots.
    - Select menu and choose 9: Construction and 5: Midpoint.
    - Cursor to the segment and press enter or click and esc to exit this menu.

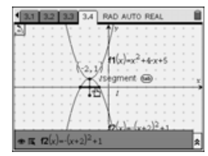




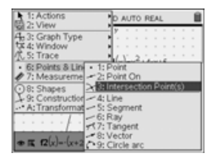
12. Rotate clockwise the segment joining the plotted complex roots about its midpoint.
    - Select menu and choose A: Transformation, 4: Rotation.
    - Select the segment, then select the center point of the rotation (segment midpoint), and then select three points that determine a clockwise rotation by 90° (top endpoint of segment, midpoint, and lower endpoint of segment) for the rotation angle.



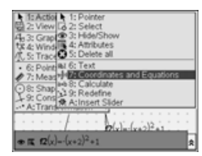
Q12: Where are the endpoints of the rotated segment joining the plotted complex roots located?
At the x-intercepts of the reflected parabola



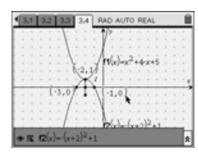
13. Locate the zeros of the reflected function (*f2(x)*).
    - Select menu and choose 6: Points & Lines and 3: Intersection Point(s).
    - Cursor to the *x*-axis, press enter, cursor to the reflected function, and press enter.
    - Press esc to return to the graph.



14. Determine the coordinates of the zeros of the reflected function.
    - Select menu, choose 1: Actions and 7: Coordinates and Equations, cursor to one of the zeros, and press enter or click twice.
    - Cursor to the other zero and press enter or click twice.



Q13: What are the coordinates of the zeros of the reflected function?
(-3, 0) and (-1, 0)



Q14: What can you conclude about the location of the roots of the function $f1(x) = x^2 + 4x + 5$ and the endpoints of the rotated segment?
They coincide.

Q15: Explain how the complex roots of a quadratic equation can be found using the graph of its related function.
If the parabola does not intersect the x-axis, reflect it vertically over its vertex. Find the x-intercepts of this reflection. Rotate these intercepts 90 degrees about their midpoint. The coordinates of these rotated points, written as complex numbers of the form a + bi and a – bi using their x-coordinate as a and their y-coordinate as b, will be the desired roots.