

Conditional Statements

ID: 8746

 Time required
 40 minutes

Activity Overview

In this activity, students will write logical statements related to the given conditional statement. They will explore whether the statements are true or false and find counterexamples for false statements. These explorations will involve the slopes of parallel and perpendicular lines and lengths of collinear and noncollinear segments.

Topic: Inductive & Deductive Reasoning

- *Write the inverse, converse, and contrapositive statements corresponding to a given conditional statement.*
- *Use a counterexample to prove that a statement is false.*

Teacher Preparation and Notes

- *This activity is designed to be used in a high school or middle school geometry classroom.*
- *Before beginning this activity, students should be familiar with the terms inverse, converse, and contrapositive.*
- *Students will discover the following concepts:*
 - *Parallel lines have slopes that are equal; perpendicular lines have slopes that are opposite reciprocals (the product of the slopes is -1).*
 - *The Segment Addition Postulate states that $AB + BC = AC$ if B is between A and C and the points are collinear. If the points are not collinear, then $AB + BC > AC$.*
- *If desired, teachers can explore which of the statements in the activity are also biconditional statements (definitions that are always true).*
- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress.*
- ***To download the Cabri Jr. files and student worksheet, go to education.ti.com/exchange and enter "8746" in the keyword search box.***

Associated Materials

- *ConditionalStatements_Student.doc*
- *COLSEG and NOCOLSEG (Cabri Jr. files)*

Suggested Related Activities

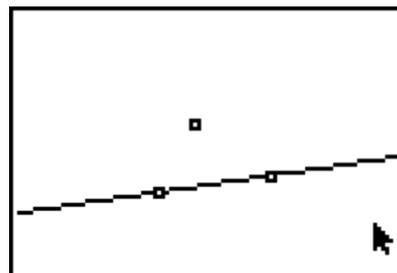
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Preparing for Proofs (TI-84 Plus family with TI-Navigator) — 7769*
- *Congruent Triangles (TI-84 Plus family) — 8817*

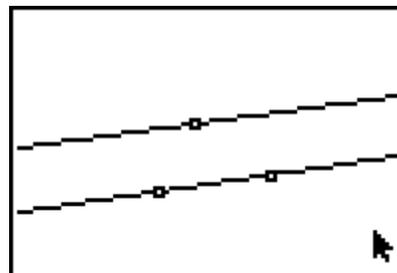
Problem 1 – Slopes of lines

To begin, students should open a new *Cabri Jr.* file.

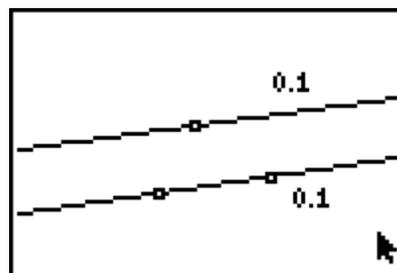
Step 1: Students will need to construct two parallel lines. First, a line needs to be constructed using the **Line** tool and a point not on the line using the **Point** tool.



Step 2: Using the **Parallel** tool, a line parallel to the existing line should be constructed through the point not on the original line.



Step 3: Students will find the slope of both lines by using the **Slope** tool (**Measure > Slope**). Students can now use the **Hand** tool to drag the original line or the point and observe the results.



What is true of the slopes of parallel lines?

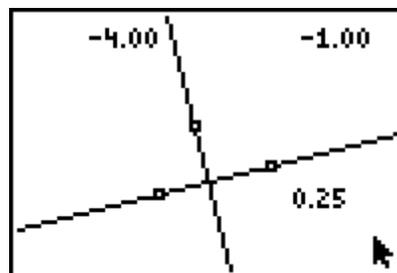
Students should record their observations on the worksheet part **A**. They will write the converse, inverse, and contrapositive of the statement and determine the truth of each.

Students will next construct perpendicular lines in a new *Cabri Jr.* file.

Step 4: Again construct a line and a point not on the line. Have students select the **Perp.** tool to construct a line perpendicular to the existing line through the point not on the original line.



Step 5: Students will find the slope of both lines. They should drag the original line or the point and observe the results.



Step 6: Have students select **Calculate** to find the product of the slopes. They should again drag the line to observe the results.

What is true of the slopes of perpendicular lines?

Students should record their observations, write conditional statements, and determine the truth of the statements on the worksheet in part **B**.

On a new Cabri Jr. file, students will construct two lines that have the same y -intercept. If the axes are not currently showing, they should select **Hide/Show > Axes**.

Step 7: Students should start by placing a point on the y -axis using the **Point > Point On** tool.

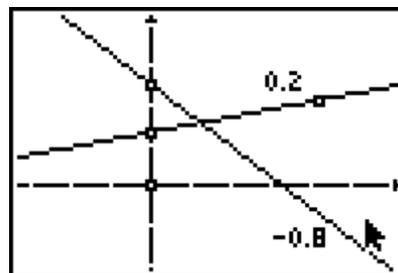
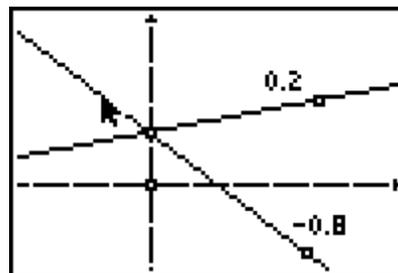
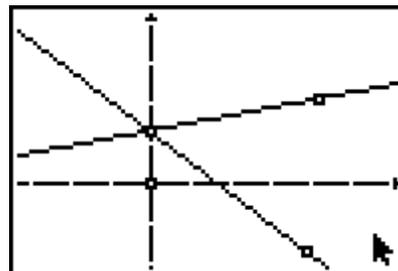
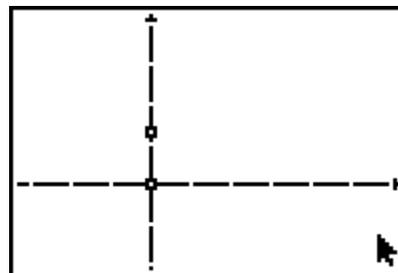
Step 8: Make sure that students construct two different lines with the same y -intercept (the point previously created on the y -axis) and find the slopes of both lines.

Note: If desired, students can display the equations of both lines using the **Coord. & Eq.** tool instead of the slopes.

Step 9: Direct students to drag the lines and the y -intercept point and observe the changes in the coordinates and slopes.

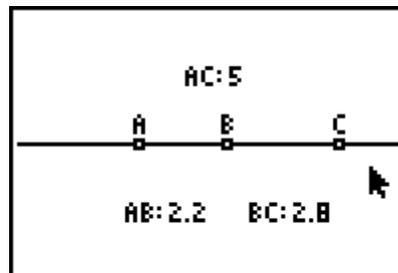
Students will write conditional statements on the worksheet part **C**. They should sketch counterexamples for any false statements.

The screenshot at right shows a counterexample for the converse statement “If two lines have different slopes, then they have the same y -intercept.”



Problem 2 – Collinear and noncollinear segments

Step 1: Distribute the file **COLSEG** to students. Points **A**, **B**, and **C** are constructed on a line and the lengths **AB**, **BC**, and **AC** are displayed.



Step 2: Students will drag the points with the **Hand** tool and observe the changes in the lengths.

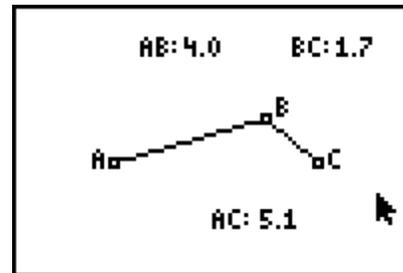
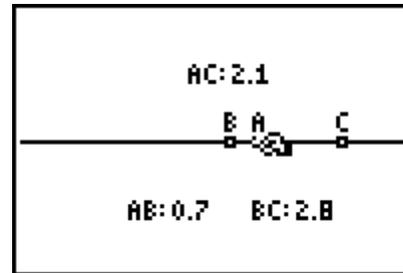
Be sure that students drag all three points, and change the order of the points (they should investigate situations where **B** is not between the other two points).

Students should record their observations on the worksheet and write a conditional statement.

Step 3: Distribute the file **NOCOLSEG** to students. Points **A**, **B**, and **C** are not collinear. The lengths **AB**, **BC**, and **AC** are displayed.

Step 4: Students should now drag the points and observe the changes in the distances. If desired, students can use the **Calculate** tool to find the sum of **AB + BC**.

Students should record observations on the worksheet and write a conditional statement.



Solutions – Student Worksheet
Problem 1

- A. If two lines are parallel, then the slopes of the lines are equal.

Converse: If the slopes of the lines are equal, then the two lines are parallel.

Inverse: If the two lines are not parallel, then the slopes of the lines are not equal.

Contrapositive: If the slopes of the lines are not equal, then the two lines are not parallel.

Each conditional statement is true.

- B. If two lines are perpendicular, then the slopes of the lines are equal to -1 .

Converse: If the slopes of two lines are equal to -1 , then the lines are perpendicular.

Inverse: If two lines are not perpendicular, then the slopes of the lines are not equal to -1 .

Contrapositive: If the slopes of two lines are not equal to -1 , then the lines are not perpendicular.

Each conditional statement is true.

- C. *Converse:* If two lines have different slopes, then the lines have the same y -intercept.

Inverse: If two different lines do not have the same y -intercept, then the lines do not have different slopes.

Contrapositive: If two lines do not have different slopes, then the different lines do not have the same y -intercept.

Each conditional statement is false.

Problem 2

- A. Answers will vary for distances of AB, BC, and AC.

The lengths AB and BC add up to equal AC when B is between A and C.

If A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.

- B. Answers will vary for distances of AB, BC, and AC.

$AB + BC > AC$

If A, B, and C are not collinear, then the sum of the lengths of AB and BC is greater than AC.