Gateway Arc Length

ID: 12440

Time Required
15 minutes

Activity Overview

*Students will investigate the arc length of the Gateway Arch. They will use the Pythagorean Theorem to approximate and use Calculus to find the exact solution. They will also use CAS capabilities, including **arcLen( )**, to solve a variety of arc length questions.*

Topic: Differential Equations

- *Arc length approximation, calculus formula, and using CAS*
- *Find the arc length of parametric equation*

Teacher Preparation and Notes

- *Arc length is a Calculus BC topic. Calculus AB teachers may enjoy using this activity after the AP* exam or with students in an AB class who want to prepare for the BC exam. After completing the activity, students should be more successful with AP questions like multiple choice 03BC15, 98BC21, 88BC#33, and free response 04formB BC1c, 02formB BC1d&3c, 01BC1c, 97BC1e&3b. For four of these six free response questions the graph is given in parametric form.*
- *The syntax for **arcLen** is **arcLen(f(x),x,a,b)** where f(x) is the function, x is the variable, and the arc length is to be found from x = a to x = b. This activity will help students approximate arc length and use calculus to find the exact arc length.*
- *Students can enter their responses directly into the TI-Nspire handheld or on the accompanying handout. On self-check questions, students can then press (menu) and select **Check Answer** (or press (ctrl) + ▲). If desired, by using the TI-Nspire Teacher Edition software, teachers can change these self-check questions to exam mode so students cannot check their answer. On any question, click the Teacher Tool Palette and select Question Properties. Change the Document Type from Self-Check to Exam.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "12440" in the quick search box.***

Associated Materials

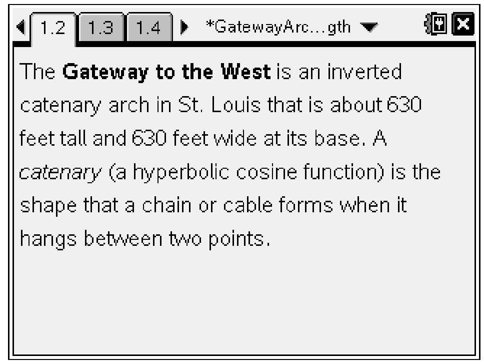
- *GatewayArcLength_Student.doc*
- *GatewayArcLength.tns*

Suggested Related Activities

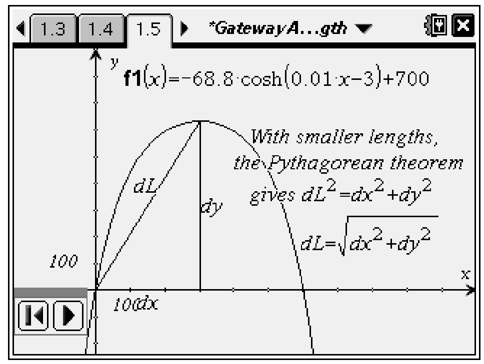
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Arc Length (TI-Nspire CAS technology) — 9897*
- *Logistic Growth, Differential Equations, Slope Fields (TI-89 Titanium) — 5514*
- *Numb3rs – Season 3 – "The Mole" – Cycloid II — 7511*

Part 1 – Arc Length Introduced

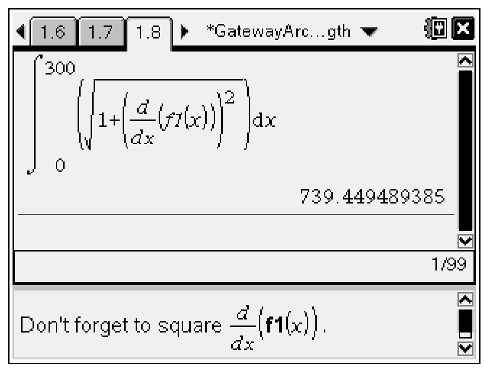


The first question investigates the Gateway Arch and the distance that one would travel if they rode the elevator tram to reach the top. The Pythagorean Theorem is used to approximate the distance. Page 1.4 helps students visually understand why those numbers where used in the solution to Exercise 1. The formula for arc length is derived from the Pythagorean Theorem. Page 1.5 shows an animation of *dx*, *dy*, and *dL* getting smaller and smaller. Students can press play to see the animation. When the infinitesimal values of *dL* are added together from *a* to *b* the arc length is found.

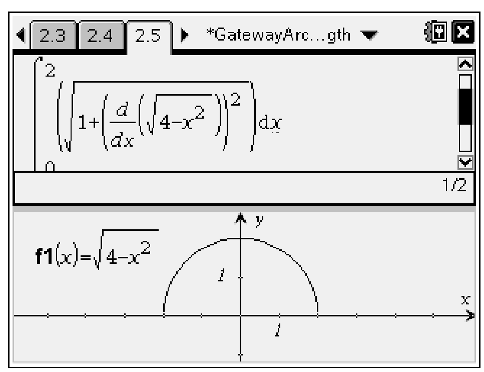


Discussion Questions

- What are the conditions for which the Pythagorean Theorem applies? *If students say, "It works for triangles," press them further.* What are characteristics of a triangle? *Perhaps they will see then, "Oh yeah, the Pythagorean Theorem only works for right triangles.* If you would like, you could go a bit deeper and ask, "What relationship (principle or law) applies for triangles that are not right? Explain it." *Law of Cosines* $c^2 = a^2 + b^2 - 2ab\cos\theta$ *where* θ *is the angle between a and b. The Pythagorean Theorem is a special case of this where* $\theta = 90°$*. (They may also say Law of Sines.)*
- Ask students, "Remind me, what is an integral? What does it mean?" *You may need to remind them that the definition was based on the area of rectangles or Riemann sums.* Ask again, "What does this mean? What are you doing with Riemann Sums?" *Adding infinite infinitesimals.*



On page 1.8, students are to use CAS to find the arc length of the Gateway Arch equation. Students compare this solution with their length from Exercise 1.



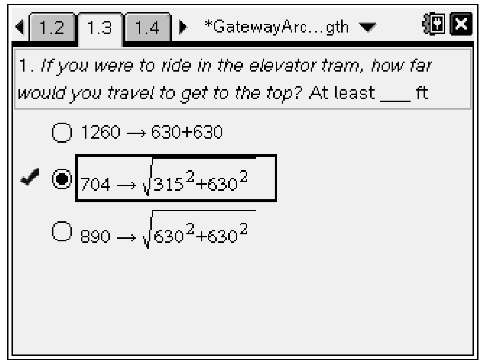
Arc length for parametric equations is introduced and students are to solve this arc length by hand. For Exercise 4, students use CAS to find the arc length for the function $y = \sqrt{4 - x^2}$ from $x = 0$ to $x = 2$. Students should make the connection that this curve is also a fourth of a circle with radius 2.

TI-Nspire Navigator Opportunity: *Screen Capture* or *Live Presenter*

See Note 1 at the end of this lesson.

Student Solutions

1. The distance is at least $704 = \sqrt{315^2 + 630^2}$. The curve will be longer than the straight line connecting the base to the peak.



2. $\int_0^{300} \sqrt{1+\left(\frac{d}{dx}(f1(x))\right)^2}\,dx = 739.449$

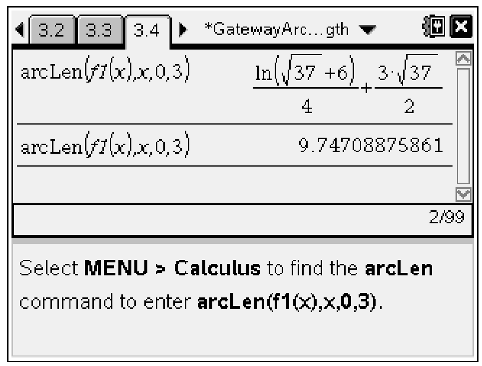
This is reasonable since it is a little larger than the straight line found in Exercise 1.

3. $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$$= \int_0^{\pi/2} \sqrt{\left(\frac{d}{dt}2\cos(t)\right)^2 + \left(\frac{d}{dt}2\sin(t)\right)^2}\,dt$$

$$= \int_0^{\pi/2} \sqrt{(-2\sin(t))^2 + (2\cos(t))^2}\,dt$$

$$= \int_0^{\pi/2} \sqrt{4\left(\sin^2(t) + \cos^2(t)\right)}dt = 2t\Big|_0^{\pi/2} = \pi$$



4. $\int_0^2 \sqrt{1+\left(\frac{d}{dx}\left(\sqrt{4-x^2}\right)\right)^2}\,dx \approx 3.14159$

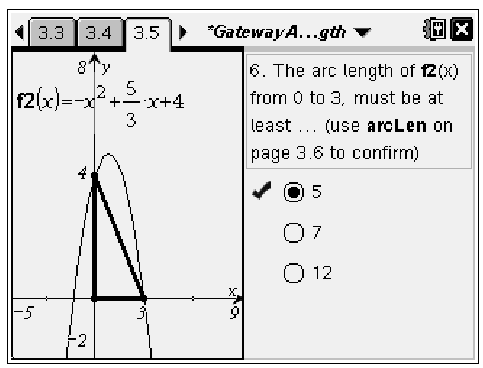
5. The solution will be around 10.

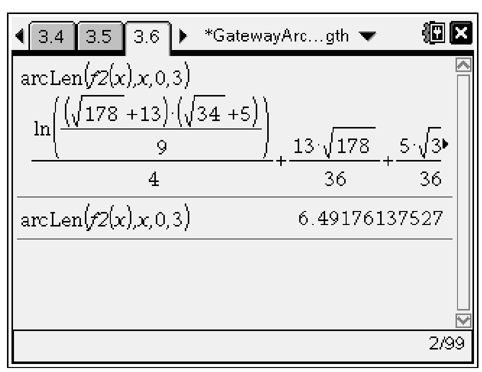
$$\int_0^3 \sqrt{1+\left(\frac{d}{dx}(x^2-9)\right)^2}\,dx \approx 9.747$$

6. The arc length will be more than 5, because $5 = \sqrt{3^2+4^2}$

$$\int_0^3 \sqrt{1+\left(\frac{d}{dx}\left(-x^2+\tfrac{5}{3}x+4\right)\right)^2}\,dx \approx 6.492$$

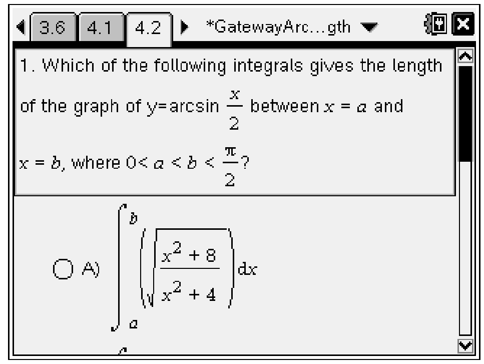
This answer is larger than 5 which is expected.

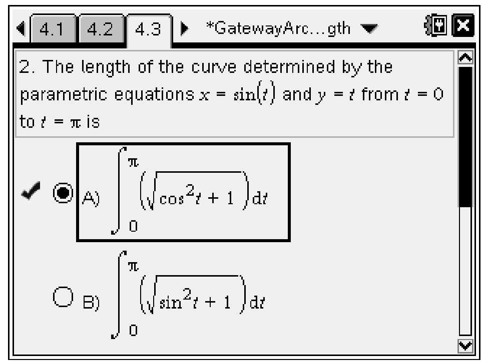


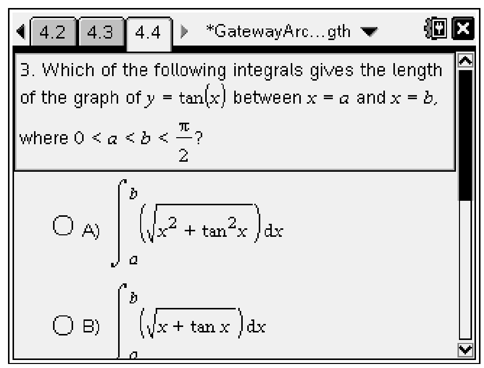


Part 2 – Additional Practice

These three exam-like questions will help students see what they are expected to know. They will be expected to know the arc length formula and answer multiple-choice questions without a calculator. Question 2 is a parametric arc length question.







Student Solutions

1. D) $\int_a^b \sqrt{\frac{x^2-5}{x^2-4}}dx$

2. A) $\int_0^{\pi} \sqrt{\cos^2 t + 1}\ dt$

3. E) $\int_a^b \sqrt{1+\sec^4 x}\ dx$

TI-Nspire Navigator Opportunity: ***Quick Poll***

See Note 2 at the end of this lesson.

TI-Nspire Navigator Opportunities

Note 1
Entire Lesson, *Screen Capture* or *Live Presenter*
Throughout the lesson, you may choose to use Screen Capture to verify students are answering the questions correctly and entering the correct formulas. You may also choose to pick one or more students as presenters to show the class how to work through the lesson.

Note 2
Entire Lesson, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.