Topics in Calculus: Applications of the Derivative

Visualizing Particle Motion: A New Look at a Classic Problem

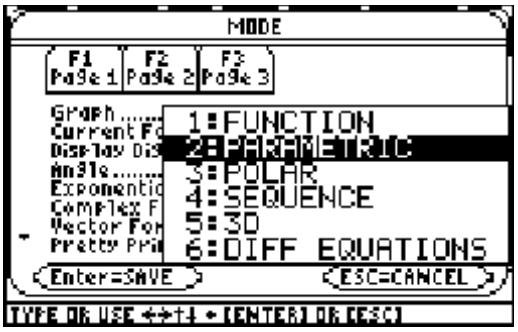
NCTM Principles and Standards

- **Content Standard:** Represent and analyze mathematical situations and structures using algebraic symbols
- **Process Standard**: Use representations to model and interpret physical, social, and mathematical phenomena

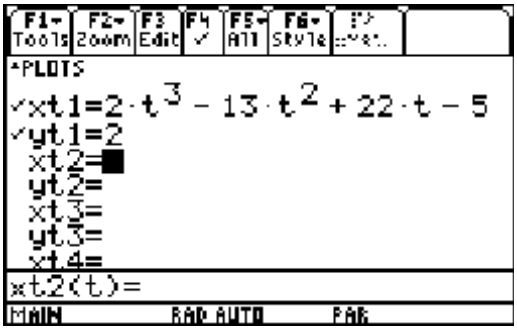
The x coordinate of a particle moving along the line y=2 is given by $s(t)=2t^3-13t^2+22t-5$ where t is time in seconds.

a. /at what time does the motion of the particle change direction?
b. Where is the particle when it changes direction?
c. What is the maximum speed of the particle for t between 1 and 3?
d. At what time is the particle at (15,2)?

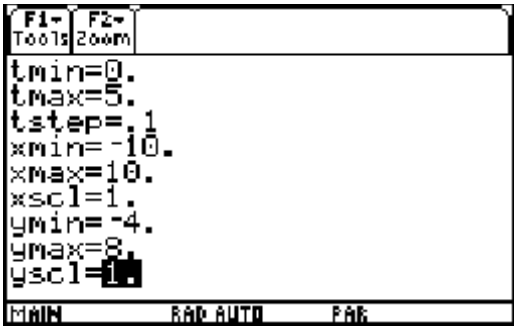
- To graph the problem first change to parametic mode by pressing [MODE] ▶ [2] [ENTER].



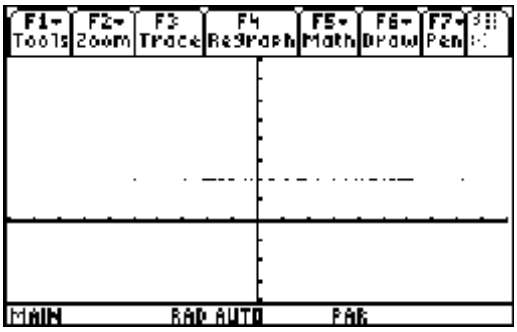
- Press [◆][F1] and enter the parametric equations for x and y as shown at the right.



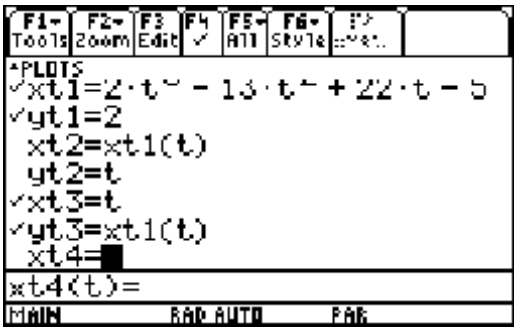
- Press [◆][F2] and set the window as shown at the right.

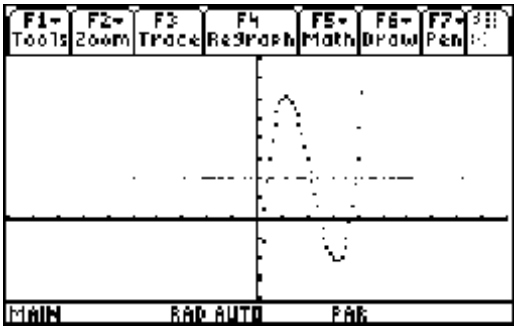


- Press ◆F3 to graph the parametric equations…not very interesting! Let's see what we can do to make the graph more meaningful.

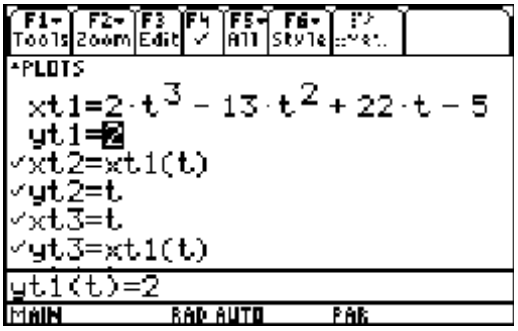


- Enter xt2, yt2, xt3, and yt3 as shown below. Press F6 (2nd F1) 2 to select the dot style. With the cursor blinking on xt2 press F4 to deselect xt2. Deselect yt2. Press ◆F3 to graph the parametric equations. The parametric equations xt1 and xt2 model the motion of the particle while xt3 and yt3 show a graph of position versus time. The distance between the dots is a measure of the rate of change.



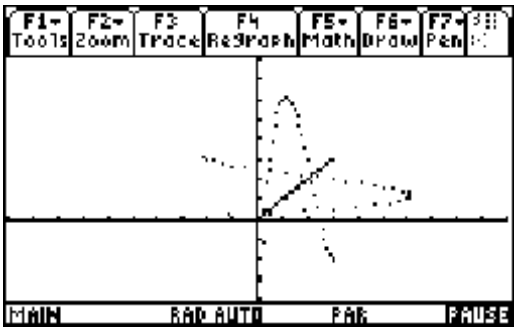


- Add the equations xt4=t and yt4=t to the y= menu and turn off xt1 and yt1 by moving the cursor to those lines in y= and pressing F4.

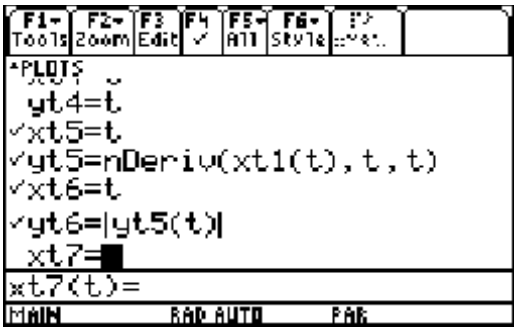


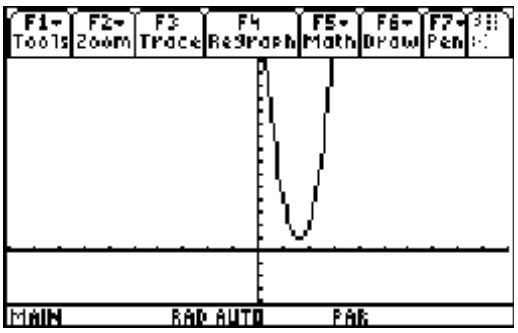
- Put the TI-89 in simultaneous mode by pressing F1 and ⊖ to go down to choice 9 (Format) on the menu or press 9 **or** simply use the shortcut keys ◆I to go directly to the format screen. Once the format screen is open press ⊖ to move down to the 2nd line and press ENTER. Select SIMUL and press ENTER.

- Press ◆F3 to graph the parametric equations. Press ENTER to pause the graph and press ENTER to resume graphing. Notice the spacing of the dots. The distance between them is the rate of change per unit of time! Press F3◀/▶ to trace the parametric equations. Press ▲/▼ to switch among the equations. Notice the inverse relationship between xt2, yt2 and xt3,yt3.



- To visualize the velocity add xt5=t and yt5=nderiv(xt1(t), t, t) to the y= menu. Let yt6=t and yt6=abs(yt5) to see the speed of the particle.





The visualization of particle motion problems along with the symbolic calculus solution will help students have a much better understanding for the dynamics of problems involving motion.