

'Poly'thagoras and Transformations

Teacher Notes

Overview

In this activity, students will explore:

- Relationships among non-square regular polygons constructed on the sides of a right triangle
- Visual and numerical proofs of the Pythagorean Theorem using rotations and non-square polygons

Supplies/Materials

TI-Nspire or TI-Nspire CAS handheld devices

Pre-requisite Knowledge and Curricular Placement

Students should have a basic understanding of regular polygons, area, the Pythagorean Theorem, and geometric transformations (rotations). Possible curricular placements include middle school, informal and regular geometry in high school, or geometry courses for pre-service or in-service teachers at the college level.

Background Information and Expectations

Oftentimes, students of all ages have memorized the formula $a^2 + b^2 = c^2$ with no real understanding of how it relates to area. For those who have seen squares drawn on the sides of a right triangle, they rarely realize that squares are not the only shape for which the Pythagorean Theorem holds. A TI-Nspire handheld device enables students to explore other similar polygons drawn on the legs of a right triangle both visually and numerically. After students complete this activity, they should possess a deeper understanding of how areas of various similar figures relate to the Pythagorean Theorem.

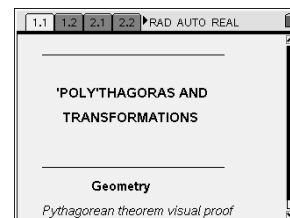
Pedagogical Suggestions

This activity is designed to be teacher led and requires minimal or no student experience with a TI-Nspire handheld device. Having students record their findings on the Student Worksheet will encourage engagement, retention, and mathematical communication. Teachers should encourage students to share their predictions and conjectures during the activity. Anticipated timing for the activity is one class period of 50 minutes.

Instructions

Open the student file Polythag.tns (teacher file Polythag_TCH.tns) on your TI-Nspire™ handheld device.

Use this document as a guide to the activity and to record your answers.

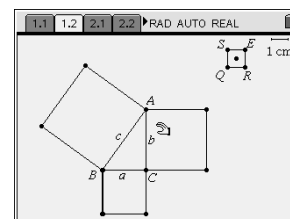


Problem 1 – Reviewing what you know

Advance to Page 1.2 by pressing  and the right side of the NavPad.

Consider right triangle ABC.

1. Explain what a^2 , b^2 , and c^2 represent.
 a^2 represents the area of the square drawn on side a ; b^2 represents the area of the square drawn on side b ; and c^2 represents the area of the square drawn on side c .



2. Explore several different right triangles by dragging a vertex and use the measurement tool to find corresponding values for a^2 , b^2 , and c^2 . Record the values below.

a^2	b^2	c^2
6.125	11.9025	18.075
9.925	11.9025	21.6599
9.6	11.9	21.5

3. What is the relationship between $a^2 + b^2$ and c^2 ?
 $a^2 + b^2 = c^2$

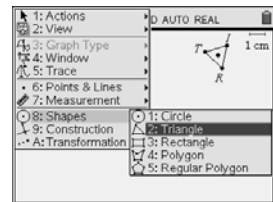
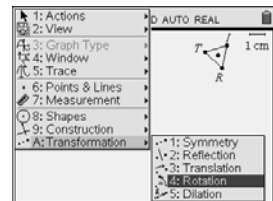
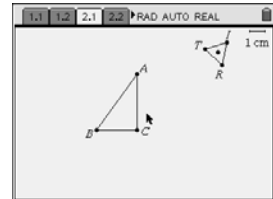
Problem 2 – Investigating equilateral triangles constructed on the sides of a right triangle

Advance to Page 2.1 by pressing ctrl and the right side of the NavPad.

4. What is the measure of each angle in an equilateral triangle such as ΔTRI ?
 60°

Step 1: Construct an equilateral triangle on side a by first rotating side a .

- Select menu and chose A: Transformation, 4: Rotation.
- Select side a , then select the center point of the rotation (point B), and then select three points that determine an angle in equilateral triangle TRI (point T , point R , and point I) for the rotation angle.
- Draw a triangle on side a using menu , 8: Shapes, 2: Triangle.



Step 2: Repeat the process in Step 1 for side b , point C , and $\angle TRI$.

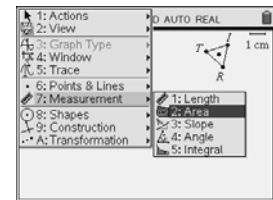
Step 3: Repeat the process in Step 1 for side c , point A , and $\angle TRI$.

5. Predict how the areas of equilateral triangles drawn on the sides of a right triangle relate.

The sum of the areas of the triangles on the legs will equal the area of the triangle on the hypotenuse.

Step 4: Find the area of each of the equilateral triangles constructed on the sides of ΔABC .

- Select menu and chose 7: Measurement, 2: Area.
- When the triangle blinks, press ctrl .



Step 5: Store the area on side a as a variable by selecting store var , 1:Store Var, and type $ar1$. Repeat this process twice more to store the area on side b as $ar2$, and the area on side c as $ar3$.

Advance to Page 2.2. Observe how the Automated Data Capture tool dynamically collects measurement data for ΔABC . Observe that your values for a^2 , b^2 , and c^2 are already listed in cells A1, B1, and C1.

area1	area2	area3	D
2.70633	5.15393	7.98026	
=2.70633+5.15393			

Step 6: To collect additional data, return to Page 2.1, drag point A around, and then drag point B around.

Step 7: Return to your spreadsheet on Page 2.2.

6. Record several of the collected triangular areas in the table below.

a^2	b^2	c^2
2.706	5.1539	7.856
6.7301	5.1539	11.884
6.7171	1.1008	7.8179

7. Make a conjecture about the relationship between $a^2 + b^2$ and c^2 for equilateral triangles constructed on the sides of a right triangle.

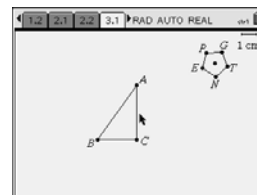
$$a^2 + b^2 = c^2$$

Problem 3 – Investigating regular pentagons constructed on the sides of a right triangle

Advance to Page 3.1.

8. What is the measure of each angle in a regular pentagon?

$$108^\circ$$

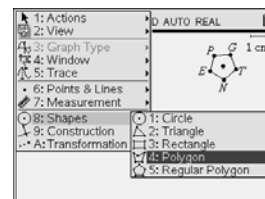


Step 8: Construct a regular pentagon on side a by first rotating side a .

- Select (menu) and chose A: Transformation, 4: Rotation.
- Select side a , then select the center of rotation (point B), and then select three points that determine an angle in pentagon PENTG (point P , point E , and point N) for the rotation angle.
- Select the rotated side, the endpoint of the segment not on the triangle, and then the three points P , E , and N , in order.
- Select the newly rotated side, the endpoint of the newly rotated side not connected to another segment, and then the three points P , E , and N , in order.
- Repeat the process one more time.
- Draw a pentagon on side a using (menu), 8: Shapes, 4: Polygon.

Step 9: Repeat the process in Step 8 for side b , initial rotation center point C , and $\angle PEN$.

Step 10: Repeat the process in Step 8 for side c , initial rotation center point A , and $\angle PEN$.



9. Predict how the areas of pentagons drawn on the sides of a right triangle relate.

The sum of the areas of the pentagons on the legs will equal the area of the pentagon on the hypotenuse.

Step 11: Find the area of each of the regular pentagons constructed on the sides of $\triangle ABC$.

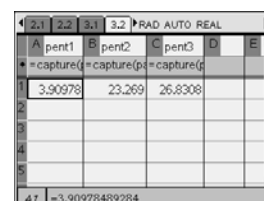
- Select (menu) and chose 7: Measurement, 2: Area.
- When the pentagon blinks, press (key).

Step 12: Store the area on side a as a variable by selecting (store Var), 1:Store Var, and type **par1**.

Repeat this process twice more to store the area on side b as **par2**, and the area on side c as **par3**.

Advance to Page 3.2. Observe how the Automated Data Capture tool dynamically collects measurement data for $\triangle ABC$. Observe that your values for a^2 , b^2 , and c^2 are again listed in cells A1, B1, and C1.

Step 13: To collect additional data, return to Page 3.1, drag point A around, and then drag point B around.



Step 14: Return to your spreadsheet on Page 3.2.

10. Record several of the collected pentagonal areas in the table below.

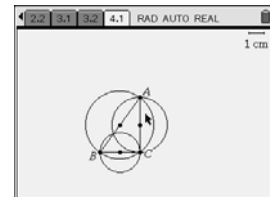
a^2	b^2	c^2
3.90978	23.269	26.8308
7.50558	23.269	30.4266
2.5064	9.64543	13.2906

11. Make a conjecture about the relationship between $a^2 + b^2$ and c^2 for regular pentagons constructed on the sides of a right triangle.

$$a^2 + b^2 = c^2$$

Extension – What happens when a non-polygon such as a semi-circle is constructed on the sides of a right triangle?

Advance to Page 4.1.



12. Record several of the non-polygon areas in the table below.

a^2	b^2	c^2
4.90874	9.3482	14.2565
4.90874	6.3794	11.2881
9.6388	10.7573	20.3961

13. Make a generalization about the relationship between $a^2 + b^2$ and c^2 for similar shapes constructed on the sides of a right triangle. Make certain to provide an explanation that supports your generalization.

$a^2 + b^2 = c^2$ since the area of any shape drawn on a leg of a right triangle added to the area of a similar shape on the other leg of a right triangle always equals the area of a similar shape on the hypotenuse of a right triangle.