LESSON

7.8

Activity Day

Decreasing Exponential Models and Half-Life

In Lesson 7.7, you learned that data can sometimes be modeled using the exponential equation $y = A(1 - r)^x$. In this lesson you will do an experiment, write an equation that models the decreasing exponential pattern, and find the **half-life**—the amount of time needed for a substance or activity to decrease to one-half its starting value. To find the half-life, approximate the value of x that makes y equal to $\frac{1}{2} \cdot A$.

Technology CONNECTION

You can see simulations of atomic half-life with a link at www.keymath.com/DA .

In the previous investigation, if your plate was marked with a 72° angle and you started with 200 "atoms," a model for the data could be $y = 200(1 - 0.20)^x$. This is because the ratio of the angle to the whole plate is $\frac{72}{360}$, or 0.20. To determine the half-life of your atoms, you would need to find out how many drops you would expect to do before you had 100 atoms remaining. Hence, you could solve the equation $100 = 200(1 - 0.20)^x$ for x using a graph or a calculator table. The x-value in this situation is approximately 3, which means your atoms have a half-life of about 3 years.

Activity

Bouncing and Swinging

You will need

- a motion sensor
- a meterstick
- a ball
- string
- a soda can half-filled with water

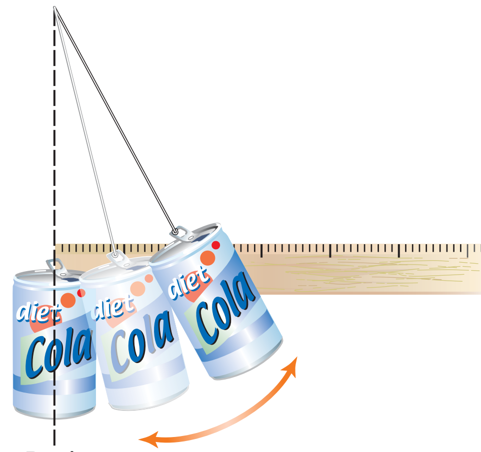
There are two experiments described in this activity. Each group should choose at least one, collect and analyze data, and prepare a presentation of results.

Step 1 Select one of these two experiments.

Experiment 1: Ball Bounce

You will drop a ball from a height of about 1 meter and measure its rebound height for at least 6 bounces. You can collect data "by eye" using a meterstick, or you can use a motion sensor. [▶ See **Calculator Note 7D** for a program to use with your motion sensor. ◀] If you use a motion sensor, hold it $\frac{1}{2}$ meter above the ball and collect data for about 8 seconds; trace the resulting scatter plot of data points to find the maximum rebound heights.

Experiment 2: Pendulum Swing



Resting position

Make a pendulum with a soda can half-filled with water tied to at least one meter of string—use the pull tab on the can to connect it to the string. Pull the can back about $\frac{1}{2}$ meter from its resting position and then release it. Measure how far the can swings from the resting position for several swings. You can collect data "by eye" using a meterstick (you may have to collect data for every fifth swing in this case), or you can use a motion sensor. [▶ See **Calculator Note 7D** for a program to use with your motion sensor. ◀] If you use a motion sensor, position it 1 meter from the can along the path of the swing; the program will collect the maximum distance from the resting position for 30 swings.

Step 2 Set up your experiment and collect data. Based on your results, you might want to modify your setup and repeat your data collection.

Step 3 Define variables and make a scatter plot of your data on your calculator. (If you used a motion sensor, you should have this already.) Sketch the plot on paper. Does the graph show an exponential pattern?

Step 4 Find an equation of the form $y = A(1 - r)^x$ that models your data. Graph this equation with your scatter plot and adjust the values if a better fit is needed.

Step 5 Find the half-life of your data. Explain what the half-life means for the situation in your experiment. (Read p. 414 to review the calculation of half-life.)

Step 6 Find the y-value after 1 half-life, 2 half-lives, and 3 half-lives. How do these values compare?

Step 7 Write a summary of your results. Include descriptions of how you found your exponential model, what the rate r means in your equation, and how you found the half-life. You might want to include ways you could improve your setup and data collection.

In the real world, eventually your ball will stop bouncing or your pendulum will stop swinging. Your exponential model, however, will never reach a y-value of zero. Remember that any mathematical model is, at best, an approximation and will therefore have limitations.