

Hanging with the Incenter

ID: 11359

Time Required

15 minutes

Activity Overview

In this activity, students will explore the angle bisector of the angles of a triangle. Students will discover that the angle bisectors are concurrent. The point of concurrency is the incenter. Students should discover the relationship between the type of triangle and the location of the point of concurrency.

Topic: Triangles & Their Centers

- Angle Bisector Theorem
- Incenter

Teacher Preparation and Notes

- This activity was written to be explored with TI-Nspire technology.
- This is an introductory activity where students will need to know how to change between pages, construct triangles, grab and move points, measure lengths, and construct the perpendicular bisector.
- The multiple choice items are self-check and students can check them by pressing + ▲.
- To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11359" in the keyword search box.

Associated Materials

- HangingWithTheIncenter_Student.doc
- HangingWithTheIncenter.tns
- HangingWithTheIncenter_Soln.tns

Suggested Related Activities

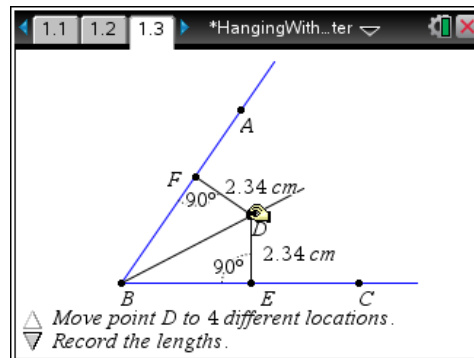
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Circumcenter and Incenter (TI-84 Plus family) — 4616
- Incenter of a Triangle (TI-84 Plus family) — 6861
- Inscribing a Circle in a Triangle (TI-84 Plus family) — 6867

Problem 1 – Exploring the Angle Bisector Theorem

On page 1.3, tell students that \overline{BD} is the angle bisector of $\angle ABC$ and that the distance from point D to \overline{BA} is the length of the segment that is perpendicular to \overline{BA} and goes through point D .

Students should find DF and DE (remember that DF means the length of \overline{DF}). The **Length** tool is found in the Measurement menu (**MENU > Measurement > Length**).



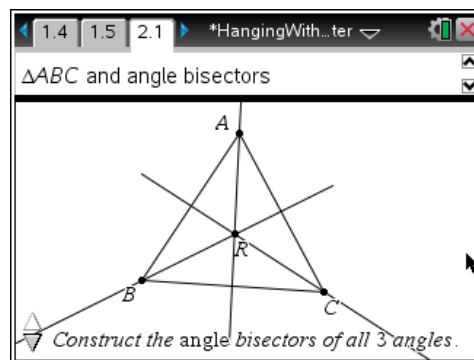
Students then will move point D to 4 different locations and record the lengths in the table on their accompanying worksheets.

Students will discover that if a point is on the angle bisector of an angle, then it is equidistant from the two segments of the angle.

Problem 2 – Exploring the Incenter

Students are to create an acute triangle on page 2.1 and find the angle bisector of all three angles of the triangle.

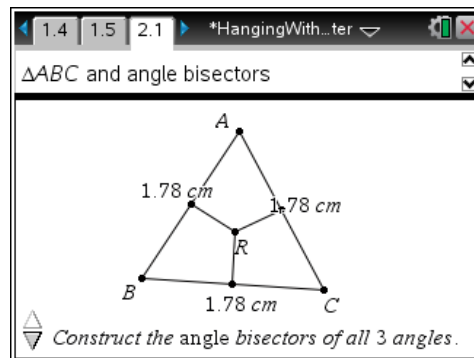
First they should select the **Triangle** tool (**MENU > Shapes > Triangle**) and click to place three points on the screen. If students press the letter of the vertex immediately after they place the point, it will label the vertex. Otherwise, students will need to use the Text tool. Then students should select the **Angle Bisector** tool (**MENU > Construction > Angle Bisector**) and the click the three points that make the angle.



Students should realize that they are concurrent and answer the questions on pages 2.2–2.5. Students can also answer Questions 4–8 on their accompanying worksheet.

Students will need to find the distance from the incenter to the 3 sides of the triangle. Students may need to be reminded how to find the distance from a point to a line or segment.

To do this, students need to use **Intersection Point(s)** tool (**MENU > Points & Lines > Intersection Point(s)**) and select the intersection of the three angle bisectors. This is a good time to hide the angle bisectors using the **Hide/Show** tool. Now they need to use the **Perpendicular** tool (**MENU > Construction > Perpendicular**) to construct a line perpendicular to the side that goes through the incenter.



Students can use the Segment tool to construct a segment from the intersection point of the perpendicular line and the side of the triangle to the incenter. Once all segments have been constructed, students should hide the perpendicular lines and use the Length tool to find the distances from the incenter to the sides.

Tell them that it is the perpendicular distance from the incenter to the side. Encourage students to hide their angle bisectors. Be sure that all students understand that the perpendicular line created from the incenter to a side of the triangle is not the perpendicular bisector.

Students should discover that all of these distances are equal.

Problem 3 – Extension

Problem 3 is an extension of this activity. Students will use the handheld and the angle bisectors to find the coordinates of fence posts on a plot of land.

Note: this problem will be very hard for the average student.

Students must first set up coordinates for the 2 fences at $(-1, 2)$, $(1, 0)$, $(1, 3)$, and $(5, 3)$. Next, students should create segments for the two fences. From here, there are several approaches to solving this problem.

Using the Incenter:

One approach is to find the incenter of the triangle formed by the intersection of the two fences (over the pond) and a segment joining the two fences.

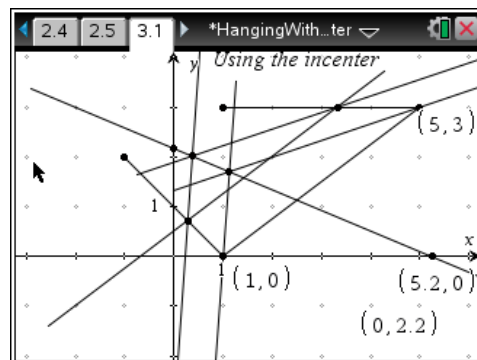
Students should construct a segment from $(1, 0)$ to $(5, 3)$ connecting the two fences. To do this, students need to select the **Segment** tool (**MENU > Geometry > Points & Lines > Segment**) and select the two points. This is to create two angles that they can use to find angle bisectors.

The point of intersection of the two angle bisectors is the incenter of the triangle. This point is therefore equidistant from the two fences.

However, two points are needed to determine the line where the fence will be.

Students will need to create a similar triangle that connects the two fences. To guarantee similarity, choose a segment that is parallel to the segment joining $(1, 0)$ and $(5, 3)$ using the **Parallel** tool (**MENU > Geometry > Construction > Parallel**).

Find the incenter of the second triangle, then create the line through those two incenters. One point on the line should be at $(5.24, 0)$ or $(5.2, 0)$, and the other can be any other point on the line connecting the two incenters. The y -intercept is another convenient point to find the coordinates of $(0, 2.2)$.



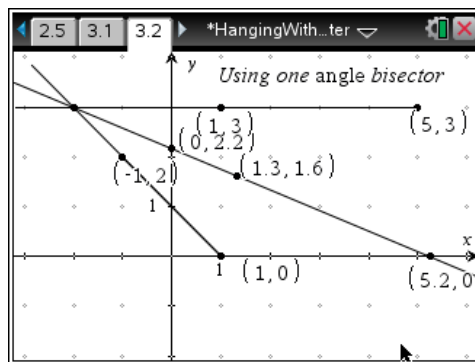
Using Only an Angle Bisector:

Another approach is to find the point of intersection of the fences (if they were continued over the pond).

Students can construct rays for the Fence 1 and Fence 2 and plot the intersection point of the rays.

Then, students can use the **Angle Bisector** tool to find the angle bisector of this angle. This ray will be equidistant from Fence 1 and Fence 2.

Students can plot a point on this ray and display and record the coordinates of this point. Then, move the point and record the new coordinates.



Student Solutions

1. Sample answers

Position	1st position	2nd position	3rd position	4th position
<i>DF</i>	2.25473	2.72284	1.85757	1.12521
<i>DE</i>	2.25473	2.72284	1,85757	1.12521

2. They are equal.
3. Equidistant
4. They are concurrent.
5. Not possible
6. Not possible
7. Obtuse, Acute, and Right
8. They are all equal.
9. Sample answer: (5.2, 0) and (0, 2.2)