Move Those Chains

Name _____

Class _____

Problem 1 – Derivative Using the Power Rule

Recall the Power Rule $\frac{d}{dx}\left(x^n\right) = n \bullet x^{n-1}$.

1. Based on the Power Rule, what do you think the derivative of $f(x) = (2x + 1)^2$ is?

Graph the derivative of the function and your conjecture about the derivative. Go to the Y= Editor. In **y1**, type **(2x+1)^2**. In **y2**, type **nDeriv(y1(x),x)**. To access the **nDeriv** command, go to the Math menu (2nd + [MATH]) and select **B:Calculus > A:nDeriv(**. In **y3**, type your conjecture for the derivative of $f(x) = (2x + 1)^2$. Highlight **y1** and press F4 to unselect this function, and press ◆ + F3 to graph **y2** and **y3**.

Note: The graphs may take a minute to appear. If the graphs of **y2** and **y3** coincide, your conjecture for the derivative may be correct. If your conjecture is incorrect, the graphs of **y2** and **y3** will not coincide.

2. Does your conjecture appear to be correct? If not, how can you change your conjecture?

3. Expand the binomial $(2x + 1)^2$. Take the derivative of each term. How does this compare with your answer to Question 1?

Problem 2 – The Chain Rule

The following are 'true' statements that can be verified on the TI-89.

$d((5x+7)\,\text{^}\,3, x) = 3 \cdot (5x+7)\,\text{^}\,2 \cdot 5x$ *true*

$d((x\,\text{^}\,3+7)\,\text{^}\,5, x) = 5 \cdot (x\,\text{^}\,3+7)\,\text{^}\,4 \cdot 3x\,\text{^}\,2$ *true*

$d((x\,\text{^}\,2+6)\,\text{^}\,4, x) = 4 \cdot (x\,\text{^}\,2+6)\,\text{^}\,3 \cdot 2x$ *true*

4. What patterns do you see? Using any information that you can infer from these statements, create a rule for finding the derivative of these functions. Discuss the patterns you see and the rule you created with a partner.

5. Using your rule from Question 4, what is $\frac{d}{dx}\left((3x+2)^2\right)$?

Verify your answer by typing your statement on the entry line of your TI-89. If you are correct, the TI-89 will return the word, 'true'. If you are incorrect, the TI-89 will return a false statement. If you are incorrect, try again by editing your statement and pressing ENTER again.

6. What is $\frac{d}{dx}\left((7x+2)^3\right)$? Verify your answer.

7. What is $\frac{d}{dx}\left((5x^2+2x+3)^4\right)$? Verify your answer.

The derivative rule you have just observed is called the **Chain Rule**. It is used to take the derivative of composite functions. The Chain Rule is $\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right) \bullet g'(x)$. First, take the derivative of the "outside function" at $g(x)$. Then, multiply this by the derivative of the "inside function."

8. Use the Chain Rule to create three additional true statements. Verify your answers.

Problem 3 – Homework Problems

Evaluate the following derivatives using the Chain Rule. Verify your answers.

1. $\frac{d}{dx}\left(\left(4x^3+1\right)^2\right)=$

2. $\frac{d}{dx}\left(\left(-5x+10\right)^7\right)=$

3. $\frac{d}{dt}\left(\left(2t^5-4t^3+2t-1\right)^2\right)=$

4. $\frac{d}{dx}\left(\left(x^2+5\right)^{-2}\right)=$

5. $\frac{d}{dz}\left(\left(z^3-3z^2+4\right)^{-3}\right)=$