Problem 1 – The second derivative of functions

The second derivative is the derivative of the first derivative.

- What is the derivative of $y = x^3 - 4x$? What is the derivative of that result?

Now, use the **Derivative** tool as follows to find the second derivative: $\frac{d}{dx}\left(\frac{d}{dx}\left(x^3 - 4x\right)\right)$.

- What answer did you get?

- How does this compare to your answer from above?

- Explain what the handheld is doing.

Use your handheld to find the second derivative for each of the following functions:

$g(x) = -x^3 + 9x$ $\quad g''(x) =$ $\quad h(x) = \cos(6x)$ $\quad h''(x) =$

$j(x) = e^{5x}$ $\quad j''(x) =$ $\quad k(x) = \frac{1}{x^2 - 1}$ $\quad k''(x) =$

Problem 2 – Concavity

The first derivative shows where the function is increasing or decreasing, but not the *shape* of the function. The second derivative is used to show the *concavity* of the function.

Graph the function $f(x) = x^3 - 4x$.

Notice that the graph for negative x-values is oriented downward. This section is *concave down*. The graph for positive x-values is oriented upward. This section is *concave up*.

Now graph both the function and its second derivative on the same set of axes.

- What do you notice about the two graphs?

- Where is the second derivative positive? Negative?

The point where the function changes concavity is called the *point of inflection*.

- What is the x-value of the point of inflection?

Use the **Tangent** tool to place a tangent line on the graph of the function. Then use the **Coordinates and Equations** tool to display the equation of the line.

	Equation of Tangent
$x = -2.0$	
$x = -1.0$	
$x = -0.5$	

	Equation of Tangent
$x = 0.5$	
$x = 1.0$	
$x = 2.0$	

- Describe what happens to the slopes as the x-values increase.

Problem 3 – Concavity for other functions

Graph $g(x) = -x^3 + 9x$ and its derivative on the same set of axes.

- What do you notice about the sign of the second derivative and the concavity of the original function?

- Where is the function concave up? Concave down?

- Is there a point of inflection on this graph? If so, where is it?

Graph the following functions and identify the intervals, if any, where the function is concave up and concave down. If there is a point of inflection, then give the point.

	Concave Up	Concave Down	Point of Inflection
$h(x) = \cos(6x)$			
$j(x) = e^{5x}$			
$k(x) = \frac{1}{\left(x^2 - 1\right)}$			