Problem 1

Rolle's Theorem: Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) such that $f'(c) = 0$.

1. Sketch the graph of a function that is continuous on an interval $[a, b]$, but does not meet the conditions of Rolle's Theorem.

2. Explain why you think it is necessary for the function to be continuous. Feel free to use sketches.

3. List the equation for $f_1(x)$ from Problem 1.

4. State the x – coordinate of A (given) and identify the x – coordinate of B (Hint: Construct the point of intersection between the horizontal line and $f_1(x)$).

5. List all the values of x between a & b (from #4) for which $f'(x) = 0$ [Hint: These values can be identified by dragging the point C along the function].

6. Using the Solve command, find all the values between a & b for which $f'(x) = 0$.

7. Compare the values from #6 to the values obtained in #5. Explain what may cause the differences.

Problem 2

Mean Value Theorem (MVT): If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there is at least one number c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

8. List the equation for $f_1(x)$ from Problem 2.

9. List the coordinates of A and B and determine the corresponding slope of $\overleftrightarrow{AB}$.

10. List all the values of x between a & b for which $f'(x)$ satisfies the MVT [Hint: These values can be identified by dragging the point C along the function]. With each value, list the corresponding slope of the tangent line.

    a. Explain any differences between the values of the slope obtained by dragging the tangent line and the target slope you listed in #9.

    b. How could you have obtained values that are closer to the target slope?

11. Using the Solve command, find all the values of x between a & b for which $f'(x)$ satisfies the MVT.

12. The Speed Trap Problem

Suppose you were driving along the interstate and passed a parked police officer going 65 mph. After 10 minutes, you pass a second parked police officer 12 miles farther down the interstate going 60 mph. The second officer pulls you over and informs you that he is going to write you a ticket for driving above the posted speed limit of 65 mph. Show how the second officer can prove you were speeding. [Hint: Consider using the MVT.]

m_{AB}

13. Prove the Mean Value Theorem.
    a. First, find the equation of the secant line which is the line through the points $(a, f(a))$ and $(b, f(b))$ in slope intercept form ($y = mx + b$).

    b. Next, define $g(x) = f(x) - y$ (from above) and simplify.

    c. Finally, use Rolle's Theorem and $g(x)$ from part b to prove the Mean Value Theorem.

Answers

The answers to 1 – 10 depend, for the most part, on the particular functions that have been assigned to individual students or groups. They should be easily determined from the .tns that you send to each student/group.

11. Convert 10 minutes to $\frac{1}{6}$ hour. Then the graph of $s(t)$, the location vs. time function, has points at (0, 0) (1st officer) and ($\frac{1}{6}$, 10) (2nd officer). The MVT states that for some c between 0 & $\frac{1}{6}$, $s'(c) = \frac{s\left(\frac{1}{6}\right)-s(0)}{\frac{1}{6}-0} = \frac{12-0}{\frac{1}{6}-0} = 72$ which proves that you were at some point in time traveling above the posted speed limit.

12a. $$m = \frac{f(b)-f(a)}{b-a}$$

$$So, \left(y - f(a)\right) = \frac{f(b)-f(a)}{b-a}(x-a)$$

$$y = \frac{f(b)-f(a)}{b-a}x - \frac{f(b)-f(a)}{b-a}a + f(a)$$

$$y = \frac{f(b)-f(a)}{b-a}x - \frac{f(b)-f(a)}{b-a}a + \frac{f(a)(b-a)}{b-a}$$

$$y = \frac{f(b)-f(a)}{b-a}x + \frac{f(a)a - f(b)a + f(a)b - f(a)a}{b-a} = \frac{f(b)-f(a)}{b-a}x + \frac{f(a)b - f(b)a}{b-a}$$

12b. $g(x) = f(x) - \frac{f(b)-f(a)}{b-a}x - \frac{f(a)b-f(b)a}{b-a}$

12c. Substitute a & b into $g(x)$

$$\begin{aligned} g(a) &= f(a) - \frac{f(b)-f(a)}{b-a}a - \frac{f(a)b-f(b)a}{b-a} \\ &= \frac{f(a)b - f(a)a}{b-a} - \frac{f(b)a - f(a)a}{b-a} - \frac{f(a)b - f(b)a}{b-a} \\ &= \frac{f(a)b - f(a)a - f(b)a + f(a)a - f(a)b + f(b)a}{b-a} = 0 \end{aligned}$$

Similarly $g(b) = 0$.

$g(a) = g(b) = 0$ means that for some c between a & b, $g'(c) = 0$ by Rolle's Theorem.

$g'(c) = f'(c) - \frac{f(b)-f(a)}{b-a} = 0$ means that $f'(c) = \frac{f(b)-f(a)}{b-a}$