Above and Beyond

Objective

Use integral calculus to investigate the relationship between the area above and below a curve.

Materials

TI-89 / TI-92 Plus / Voyage 200

Author

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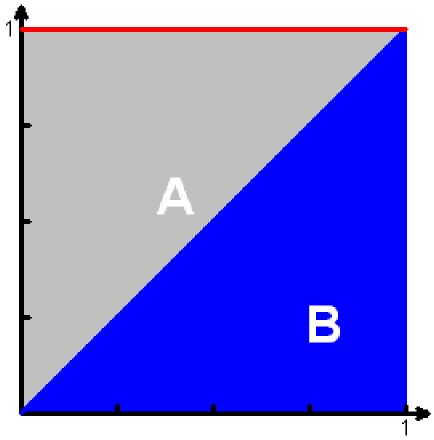
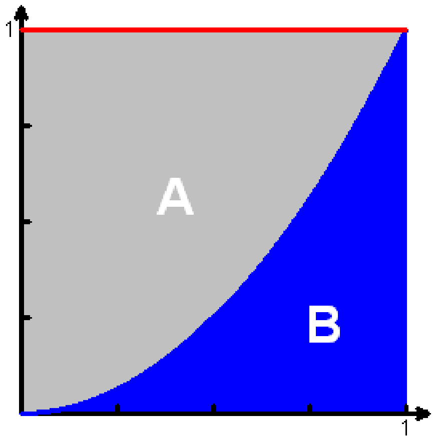
Preliminary Investigation:

1) Given the graphs of $y = x$, $y = 1$ and $x=1$, what is the ratio of Area A to Area B?
   Area A and B are shown in the diagram opposite.



2) This problem becomes more interesting by considering the graphs: $y = x^2$, $y = 1$ and $x = 1$. What is the ratio of Area A to Area B?



3) Repeat this process for $y = x^n$ where $n = \{3, 4, 5\ldots\}$ with a view to generalising the ratio of Area A to Area B.

4) Use integral calculus to determine a general rule for area B.

5) Determine an expression for Area A and hence the value of the ratio: Area A: Area B

Further Exploration:

The ratio between areas A and B can be further explored using a computer algebra system. To begin with the previous answers can be checked using some ‘clever’ function definitions.

6) Define $below(n) = \int_0^1 x^n\,dx$ Use this definition to check the areas for n = 1, 2 and 3.

7) Write down your definition for a function called ‘above’.

8) Check your ratios for n = 1, 2 and 3 using the functions *below(n)* and *above(n)*

9) What would you predict for the general case: $\frac{above(n)}{below(n)}$?

10) What does your CAS return for the general case? Explain.

The general formula has been established for positive integer values of n. The power of a general formula is that it allows us to explore other possibilities.

11) Does the formula work for negative integer values?
ie: *What happens if* n = -2? Show whether your prediction is right or wrong?

12) Does the general formula work for any other values of n? (ie: n = 1.5?)