Hit the brakes, Charles!

Teacher Notes/Answer Sheet

7 8 9 10 **11** **12**

TI-Nspire Investigation Teacher 60 min

Introduction

Hit the brakes, Charles! – bivariate data including x^2 data transformation.

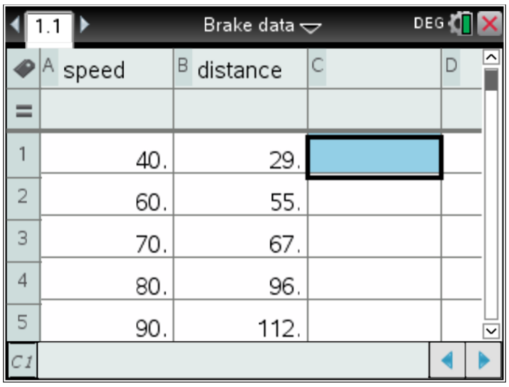
As soon as Charles got his P's he did some real data collection for his mathematics project. He decided to check whether the Auto Club figures for stopping distances and speed were true.

The table below shows the speed in *km/h* when Charles hit the brakes with the distance in *m* it took him to come to a complete stop.

Speed km/h	40	60	70	80	90	110	120
Stopping distance m	29	55	67	96	112	168	223

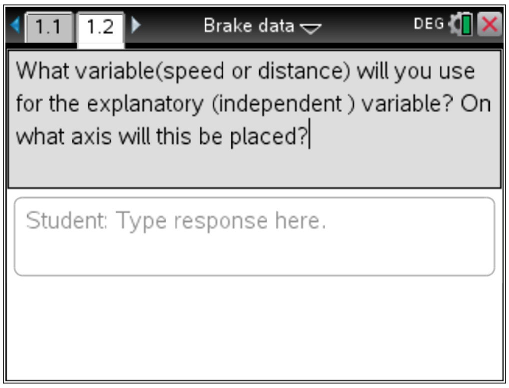
Problem 1A

Construct a data table in the **Lists & Spreadsheet** page using the variables ***speed*** and ***distance***.



Pose questions to test key knowledge in a **Notes** page or insert a **Question**.

e.g. What variable (***speed*** or ***distance***) will you use for the explanatory (independent) variable? On what axis will this be placed?



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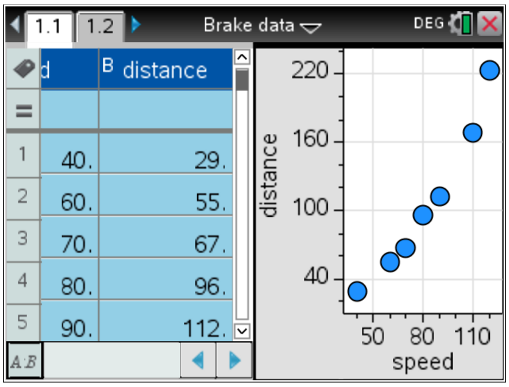
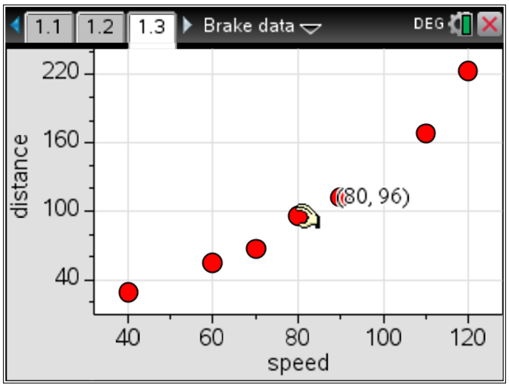
Problem 1B

Construct a scatter plot showing the relationship between the speed and braking distance. This can be plotted directly in the **Data & Statistics** page or alternatively highlight the two data columns and use menu **>Data>Quick Graph** to plot the data. Highlight column A by pressing cursor up arrow ▲ until highlighted, hold down the shift button and press the right cursor arrow ▶ to include column B. Although accessed from the **L&S** page, this method automatically uses the **Data & Statistics** page.

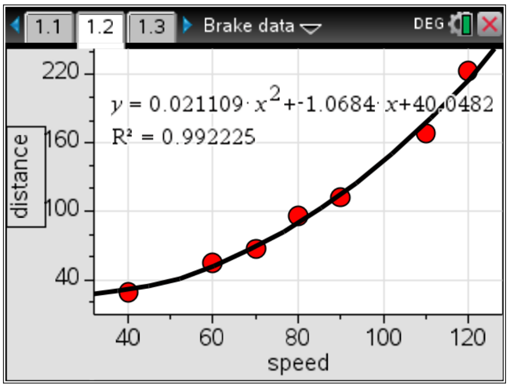


By inserting the **Data & Statistics** page directly a full screen version can be shown. In this case it is not necessary to highlight the columns. You will need to mouse click on the axes to insert the variables.



In some courses a suitable regression may be performed now with no further data manipulation and questions asked based on the regression equation obtained. Others will require data transformation to form a linear relationship (covered in following sections).

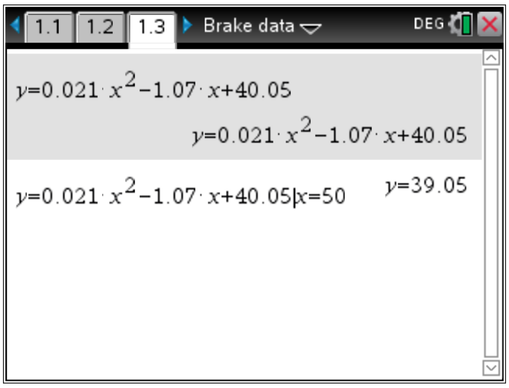
e.g. a quadratic regression shown here.



Use the regression equation to predict the stopping distance of a car travelling at 50 km/h.

Answer: Approx 39 m.

Compare the answer to the one obtained later after using a data transformation.



Note: If you are asked to do a data transformation to linearise (e.g. VCE Further Mathematics) then this answer is not acceptable. Instructions on this procedure follows. For courses not requiring this technique students can be given the transformed data to continue as a linear relationship.

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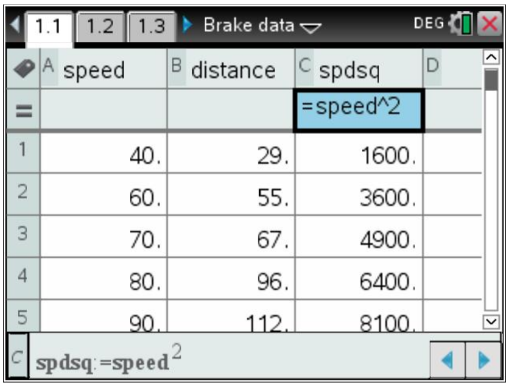
Problem 2A

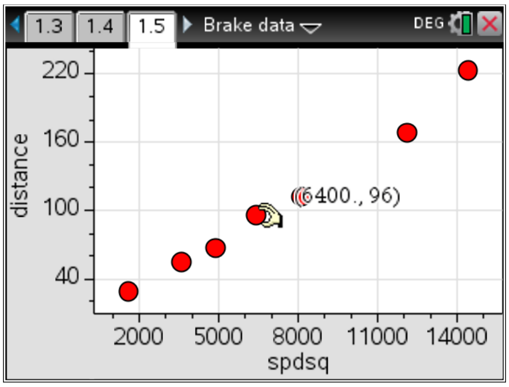
Apply an x^2 transformation using a formula in the **Lists & Spreadsheet**.

Use ctrl + ◄ to return to the **Lists & Spreadsheet** and use a formula as shown. Label the list as ***spdsq***.



Hint: use the var key to paste in list variables to avoid any typing errors.

Construct a scatter plot on a new **Data & Statistics** page showing the relationship between y (distance) and x^2 (speed squared). You will need to mouse click on the axes label to insert the variables.



To go to other existing pages in your document at any time press ctrl + ▲ to see thumbnail pages. Use your cursor to highlight the page you want and press enter. If you want the previous page you can also use ctrl + ◄, similarly ctrl + ► will give the next page.

Problem 2B

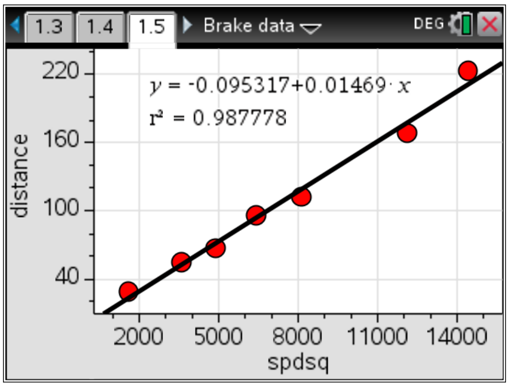
Determine the least squares regression for this transformation.

On the linearised scatter plot page press

menu **>Analyze>Regression>Show Linear (a+bx)**.

Note: in statistics (and in many course structures and exams) the preferred linear form is a+bx. You may prefer to use the more familiar form of mx+b. Make sure you let the students know which one to use.

Hint: to show the r^2 value when doing linear regressions press menu **>Settings** and tick the **Diagnostic** field, then select **Make Default**.



Answer: correct to 3 d.p.

y(distance)=-0.095 + 0.015x (speed squared)

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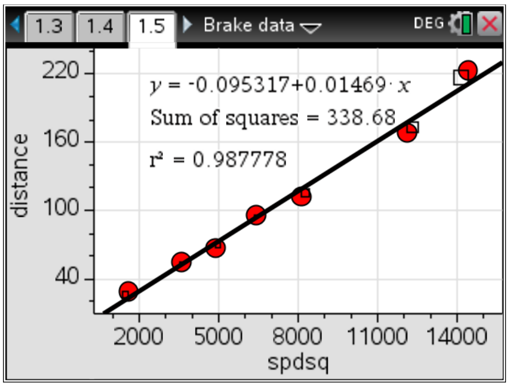
By pressing menu **>Analyze>Residuals>Show Residual Squares** you can display the residual squares.

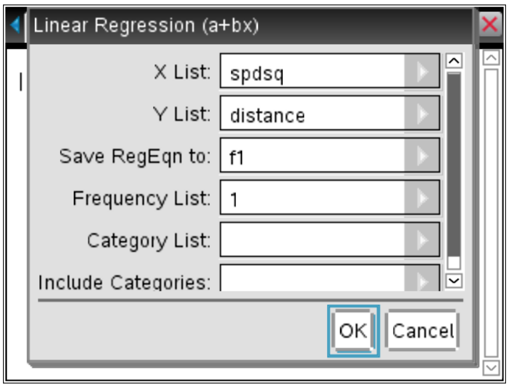


Note: You can also add your own line of best fit. Press menu **>Analyze>Add Movable Line**.

What is the meaning of the **Sum of squares** in relation to the Linear regression line?

Answer: Discuss in terms of the Least Squares Regression Line.

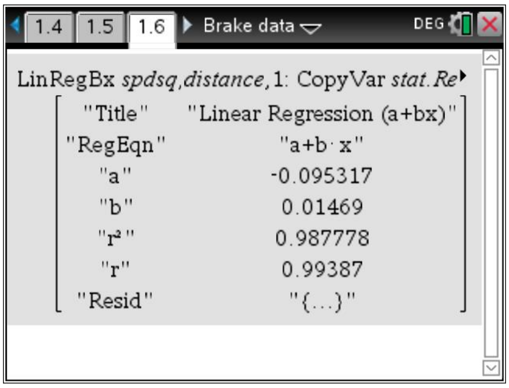
A full statistical display can be shown in a **Calculator** page using menu **>Statistics>Stat Calculations>Linear Regression (a+bx)**.



*Note: if you have performed a linear regression in the **Data & Statistics** page you can access the statistics in the **Calculator** page by pressing* var ***>Stat Results**.*

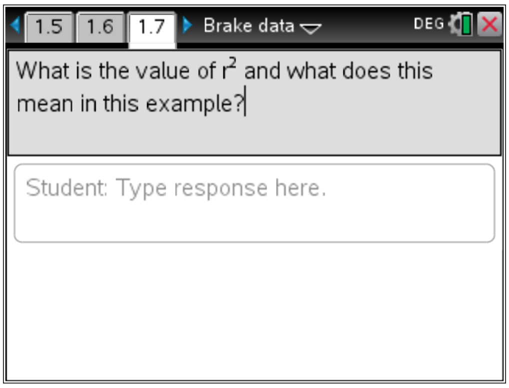
The regression equation y=-0.0953+0.0147x in this case is:

distance= -0.0953 + 0.0147 *speed squared*



What is the value of r^2 and what does this mean in this example?

Note: many exam questions are of this type.



Answer: The coefficient of determination tells us that 99% of the variation in the distance (the dependent or response variable) is explained by the variation in the square of the speed (the independent or explanatory variable).

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Problem 2C

Show a residual plot (residual vs speed squared) to verify that the x^2 transformation linearised the data.

On **D&S** page showing the transformed data plot with regression line press menu **>Analyze>Residuals>Show Residual Plot**.

Does the residual plot show any obvious pattern?

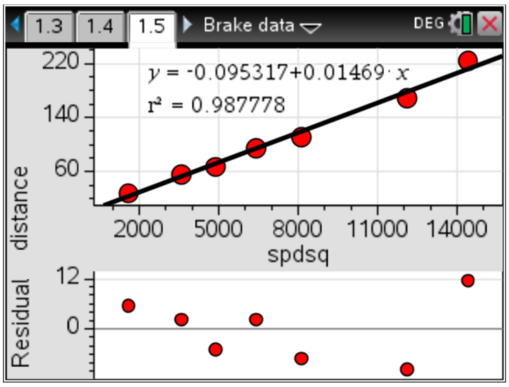
For a data transformation to be considered appropriate it must satisfy two things.

1. ***r^2 value was close to 1***

2. ***No obvious pattern in residuals***

Was the x^2 transformation appropriate in this example?

Yes



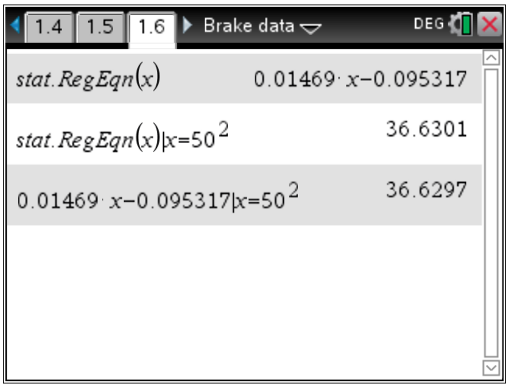
Answer: a transformation is considered appropriate if the r^2 value is close to 1 (some authors say high is $0.85 \le r^2 \le 1$) and there is no pattern in the residual plot. In this example the x^2 transformation was appropriate.

Note: you can create a separate residual plot but is best done as a split display to show patterns compared to original plot.

Use the regression equation to predict the stopping distance (to the nearest metre) of a car travelling at 50 km/h

In the **Calculator** page recall the regression equation by pressing var and select **stat.regeqn** to paste to work area.

You need to add (x) after pasting as shown. When substituting in the speed value remember that the x now represents *speed squared* so enter as 50^2.



Answer: 37 m (to the nearest metre)

The way this is calculated will depend on the skills of students. It may be appropriate to substitute directly into a rounded equation but be aware of the required precision in the question.

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