Activity 11

Objective

- To explore visual proofs of the Pythagorean Theorem using area and transformations

Cabri® Jr. Tools

Proof of The Pythagorean Theorem Using Transformations

Introduction

The Pythagorean Theorem is used in both algebra and geometry. It has been proven in more than one hundred ways, but using area and transformations is one of the most visual. In this activity, you will explore visual proofs of the Pythagorean Theorem. Since the classic representation of the Pythagorean Theorem is used in this activity, techniques for constructing a square are needed to complete the figure.

This activity makes use of the following definitions:

Hypotenuse — the side of a right triangle opposite the right angle

Legs — the sides of a right triangle that form the right angle

Part I: Exploring the Area Relationship

Construction

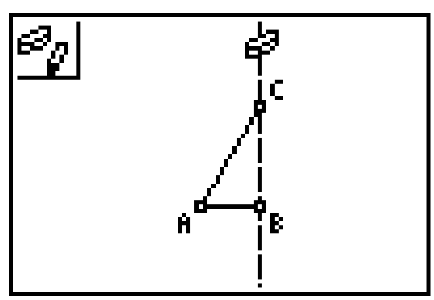
I. Construct a right triangle.

Draw a segment $\overline{AB}$.

Construct a line $\overleftrightarrow{BC}$ that is perpendicular to $\overline{AB}$.

Construct $\triangle ABC$.

Hide $\overleftrightarrow{BC}$.



Note: Verify that you have constructed a right triangle by dragging the sides and vertices to ensure that the properties of a right triangle are preserved.

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II. Construct the square of side $\overline{AB}$ using a rotation and a translation.

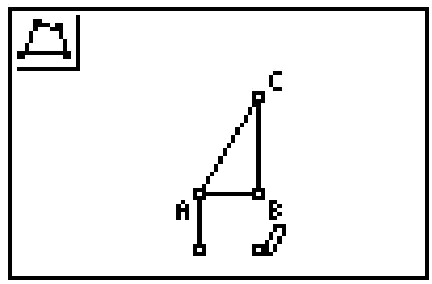
Rotate side $\overline{AB}$ 90º in a clockwise direction around point *A*.

- Select what is to be rotated (side $\overline{AB}$ in this example).
- Select a point to be the center of rotation (point *A* in this example).
- Identify the angle of rotation by selecting three points that define an angle (points *A*, *B*, and *C* in this example).

Translate point *B* using the endpoints of the rotated segment.

- Select the item to be translated (point *B* in this example).
- Select the translation segment by selecting the endpoints of the rotated segment starting with the endpoint that is also a vertex (point *A* in this example).

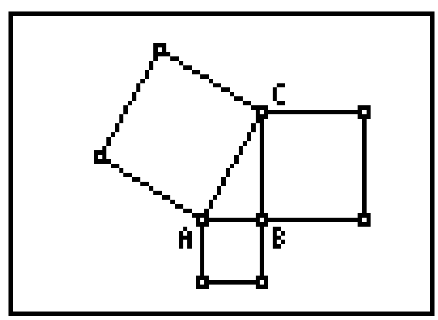
Construct the quadrilateral containing $\overline{AB}$, the image of the rotation, and the image of the translation.



Note: *Change the length of side $\overline{AB}$ to verify that the quadrilateral constructed a square based on side $\overline{AB}$.*

III. Construct the square of sides $\overline{BC}$ and $\overline{CA}$.

Repeat Construction step II for sides $\overline{BC}$ and $\overline{CA}$.



Note: *You may want to save your figure, using the name PYTHAGOR. It will be used in Part II of this activity.*

Exploration

Measure the areas of the three squares. Change ΔABC by dragging each of the vertices. Notice any relationships that exist among the areas of the three squares.

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Questions and Conjectures

1. Explain how the construction steps used to create a square guarantee that a square is actually constructed. Be prepared to provide an oral or written argument.
2. Make a conjecture about the relationship between the areas of the squares and the sides of a right triangle.

Extension

Explore the relationships among other regular polygons constructed on the sides of a right triangle.

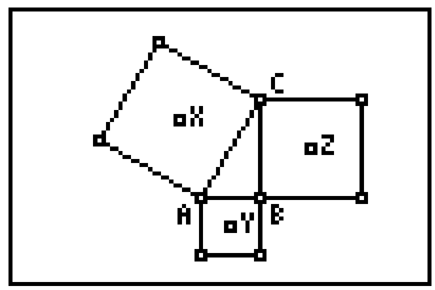
Part II: A Visual Proof of the Pythagorean Theorem

Construction

I. **Find the midpoints of the three squares.**

Clear any measures or open the Cabri® Jr. file, PYTHAGOR.

Use the **Midpoint** tool to find the midpoint between two opposite vertices of each square.

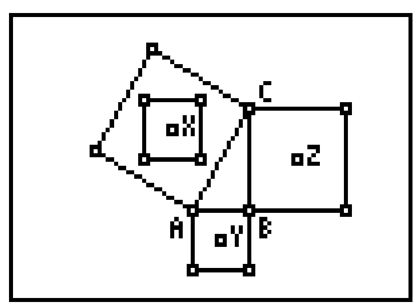


Note: The midpoints for the squares are labeled only for clarity in the construction. Do not label the midpoints.

II. **Translate the smallest square.**

Translate the square constructed on the shortest leg into the square constructed on the hypotenuse.

- Select the smallest square (Square with center *Y* in this example) as the object to translate.
- Select the center of the smallest square (*Y* in this example) as the initial endpoint of the translation segment.
- Select the center of the largest square (*X* in this example) as the final endpoint of the translation segment.



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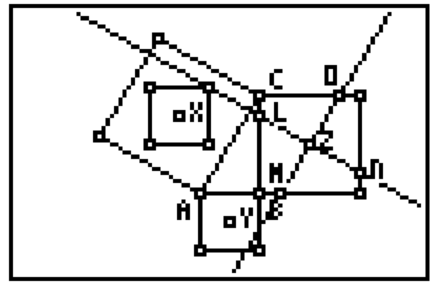
III. Divide the third square.

Construct a line parallel to side $\overline{AC}$ passing through the center of the third square (*Z* in this example).

Construct a line perpendicular to side $\overline{AC}$ passing through the center of the third square.

Construct points *L*, *M*, *N*, and *O* where the two lines intersect the third square.



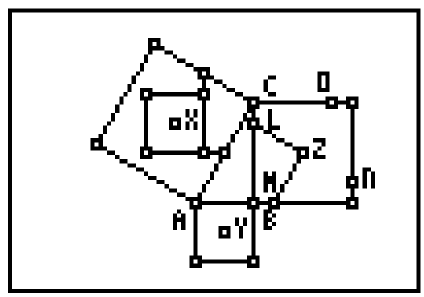
IV. Move a piece of the third square into the largest square.

Hide the parallel and perpendicular lines used in the construction.

Construct the quadrilateral *BMZL* as shown in the figure.

Translate the new quadrilateral into the largest square.

- Select quadrilateral *BMZL*.
- Select the center of the third square. (*Z* in this example.)
- Select point *C*.



Exploration

Use additional transformations to verify that this construction supports the Pythagorean Theorem. Be sure to drag the vertices of $\triangle ABC$.

Questions and Conjectures

Explain how the construction validates the Pythagorean Theorem. Be sure to discuss the additional transformations necessary to come to this conclusion. Under what conditions would this construction *not* work?

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Teacher Notes

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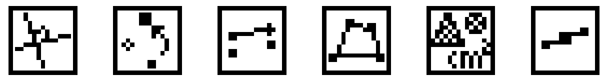

Part I: Exploring the Area Relationship

Answers to Questions and Conjectures

1. Explain how the construction steps used to create a square guarantee that a square is actually constructed. Be prepared to provide an oral or written argument.

   The squares are formed by using a rotation and a translation of one of the sides. The 90º rotation ensures that the two sides are perpendicular and equal in length. By translating Point B the length of $\overline{AB}$ and in the direction of the rotated segment, it ensures that all four sides are of equal length.

2. Make a conjecture about the relationship between the areas of the squares and the sides of a right triangle.

   The students should discover that the sum of the areas of the two squares formed on the two legs is equal to the area of the square formed on the hypotenuse.

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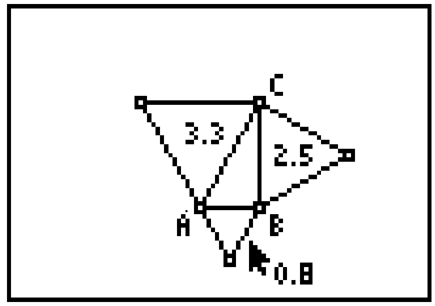
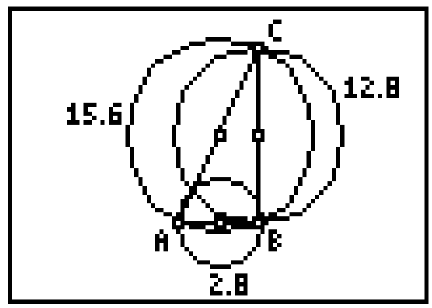
Answer to Extension

Explore the relationships among other regular polygons constructed on the sides of a right triangle.

The sum of the areas of the two regular polygons constructed on the legs of a right triangle equals the area of the regular polygon constructed on the hypotenuse. For an equilateral triangle (shown), a rotation angle of 60° is used. For other regular polygons, the rotation angle will be 180 – 360/*n*, where *n* is equal to the number of sides.



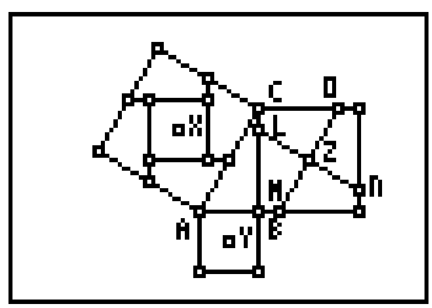
Although a circle is not a polygon, this relationship also holds true for circles that have diameters equal to the lengths of the sides of the triangle.



Part II: Proof using Translations

Answers to Questions and Conjectures

Explain how the construction validates the Pythagorean Theorem. Be sure to discuss the additional transformations necessary to come to this conclusion. Under what conditions would this construction *not* work?



Students should be able to show that the area of the smaller square and the four congruent quadrilaterals formed in the square of the larger leg will fit exactly inside the square formed by the hypotenuse (the largest square). The perpendicular lines drawn in the larger square guarantee that each of the 4 congruent quadrilaterals have one right angle that fits precisely into each of the corners of the largest square. The orientation of the translation is fixed by making the first line parallel to the hypotenuse of the triangle. One way to show that this construction works is to create and translate each of the other three quadrilaterals into the largest square. This construction will not work if the square of the larger leg is translated as a whole into the largest square.

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