# Transformers

ID: 8776

Name ______________________

Class ______________________

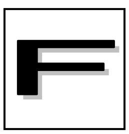
*In this activity, you will explore:*

- *Reflecting and rotating polygons*
- *Multiplying matrices to transform polygons*
- *Applying multiple transformations to a polygon*

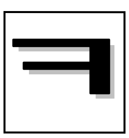

To start, open the program **TRANSFOR** found in the **Programs** menu.

## Problem 1 – Symmetry group for a square

### Identity

| Sketch | Description | Inverse |
|---|---|---|
|  | no change | no change |

### Reflections

| Sketch | Description | Inverse |
|---|---|---|
|  | reflect over $x = 0$ | reflect over $x = 0$ |
| | reflect over $y$ = ____ | |
| | reflect over $y$ = ____ | |
| | reflect over $y$ = ____ | |

**Rotations**

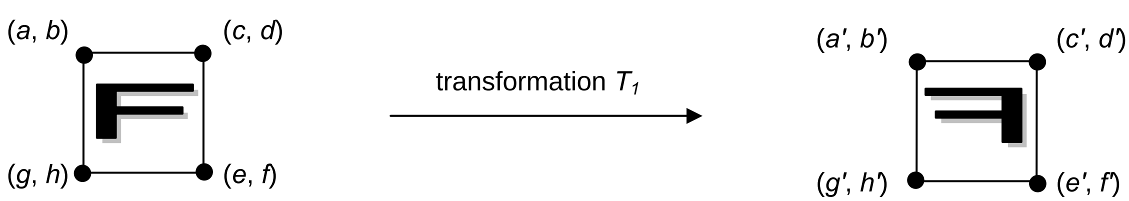
| Sketch | Description | Inverse |
|---|---|---|
| | rotate around origin ____ ° | |
| | rotate around origin ____ ° | |
| | rotate around origin ____ ° | |

- How many different transformations are in the symmetry group of a square? Include the identity.

- What do you notice about the inverse transformations? Describe them.

**Problem 2 – Transformer matrices**

**original square *S*** **image square *S′***



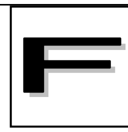
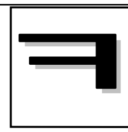

$$\mathbf{S} \times \mathbf{T_1} = \mathbf{S'}$$

$$\begin{bmatrix} a & b \\ c & d \\ e & f \\ g & h \end{bmatrix} \times \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -a & b \\ -c & d \\ -e & f \\ -g & h \end{bmatrix}$$

- Find ***S****T2*. (***T2*** is given below).

- What transformations could T2 correspond to?

Complete the table.

| **Transformer Matrix** | **Sketch** | **Description** |
|---|---|---|
| $T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ |  | no change |
| $T_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ |  | reflect over $x = 0$ |
| $T_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ | | |
| $T_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ | | |
| $T_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ | | |
| $T_5 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ | | |
| $T_6 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ | | |
| $T_7 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ | | |

Use the description columns to match the transformer matrices with their inverses. For example, $T_1$ is its own inverse.

| **Transformer Matrix** | **Inverse** | **Transformer Matrix** | **Inverse** |
|---|---|---|---|
| $T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | | $T_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ | $T_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ |
| $T_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ | | $T_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ | |
| $T_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ | | $T_5 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ | |
| $T_6 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ | | $T_7 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ | |

- Multiply each transformer matrix in the table above by its inverse.
  What do you notice?

**Use matrix multiplication to answer each question.**

- What is the effect of applying $T_3$ followed by $T_5$?

- What is the effect of applying $T_2$ followed by $T_3$?

**Problem 3 – Symmetry group for an equilateral triangle**

Use these transformer matrices.

$$T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad T_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \quad T_3 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

| Sketch | Description | Inverse | Transformer Matrix |
|---|---|---|---|
| | | | $T_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ |
| | | | |
| | | | |
| | | | |