The Pythagorean Theorem—and More

ID: 8287

Time required
30 minutes

Activity Overview

In this activity, students construct a triangle and find all angle and side measures. They practice dragging the vertices to form certain types of triangles, and then they confirm the Pythagorean Theorem for right triangles: $a^2 + b^2 = c^2$. Moreover, they discover the types of triangle that occur when $c^2 < a^2 + b^2$ or when $c^2 > a^2 + b^2$.

Topic: Right Triangles & Trigonometric Ratios

- *Construct and measure the side lengths of several right triangles and conjecture a relationship between the areas of the squares drawn on each side.*
- *Prove and apply the Pythagorean Theorem.*

Teacher Preparation and Notes

- *This activity is designed for a high school geometry classroom. It may be used to either introduce or review the Pythagorean Theorem. This activity is intended to be primarily **teacher-led**, with breaks for independent student practice. Use the following pages with the student worksheet to present the material to the class and encourage discussion. Students will follow along using their handhelds. When students are working independently, be sure to walk around the room and help as needed.*
- *This activity assumes prior knowledge of acute, right, and obtuse angles and triangles, as well as isosceles and scalene triangles.*
- *Information for an optional extension is provided at the end of this activity. It is provided on the student worksheet and in Problem 3 of the student .tns file. Should you not wish students to complete the extension, you may delete Problem 3 of the .tns file and have students disregard that portion of the student worksheet.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8287" in the keyword search box.***

Associated Materials

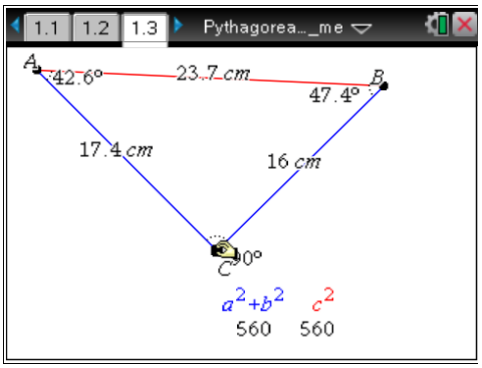
- *PythagoreanTheorem_Student.doc*
- *PythagoreanTheorem.tns*

The Pythagorean Theorem is named for Pythagoras, a Greek mathematician who lived about 2500 years ago, although there is evidence that the result was known and used prior to his time.

Problem 1 – Investigating side lengths

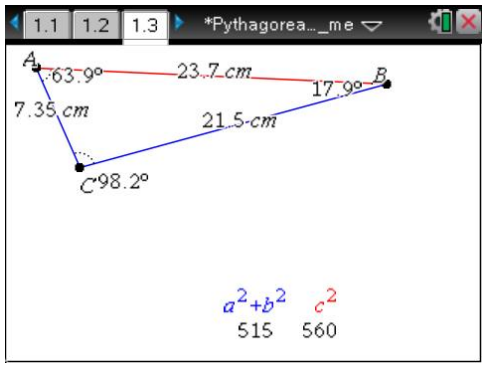
Step 1: Allow students to drag the vertices of the triangle to create several different types of triangles, such as an acute isosceles, acute scalene, right isosceles, and a right scalene triangle. When students are finished exploring their triangle, they should create a right triangle, where $\angle C$ is the right angle.

Explain that in a right triangle, c is usually used to represent the hypotenuse while a and b are usually used to represent the legs. Side c is opposite $\angle C$ and so on.



Step 2: Discuss the results of the calculated values, $a^2 + b^2$ and c^2. Base the level of discussion on your students' previous experience with the Pythagorean Theorem.

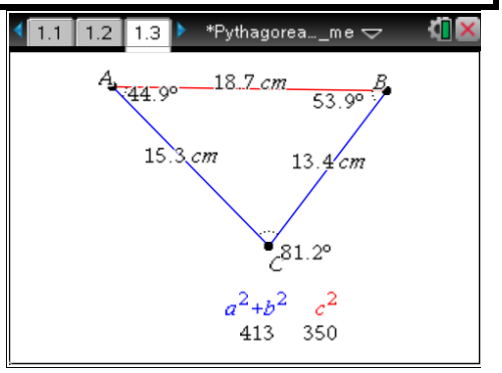
(Due to rounding, displayed values may be close, but not exact.)



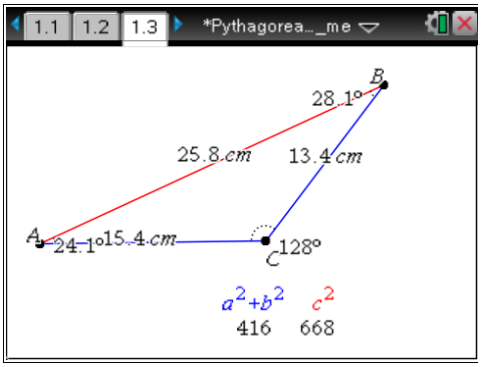
TI-Nspire Navigator Opportunity: *Quick Poll* and *Screen Capture*

See Note 1 at the end of this lesson.

Step 3: Now students can move vertices A or B so that the triangle becomes an acute triangle and compare the new values of c^2 and $a^2 + b^2$ as they do so. Encourage them to observe that as long as the triangle is acute, then $c^2 < a^2 + b^2$.



Step 4: Students should now move vertices *A* or *B* in the *opposite* direction so that the triangle becomes an obtuse triangle. Again, have them to compare the values of c^2 and $a^2 + b^2$ as they do so. They will see that as long as the triangle is obtuse, then $c^2 > a^2 + b^2$.

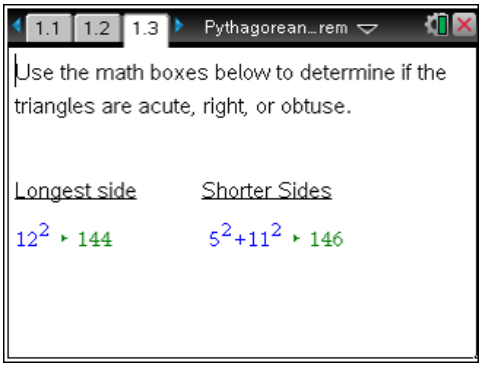


TI-Nspire Navigator Opportunity: *Quick Poll* and *Screen Capture*

See Note 2 at the end of this lesson.

Step 5: Students can use interactive Notes application on page 1.3 to complete the problem set on their worksheet. On the left, they can enter the values of c^2, and on the right, they can find the corresponding values of $a^2 + b^2$.

To evaluate expressions, press enter while in the math box.



Because the hypotenuse is always the longest side of a right triangle, have students always use *c* for the longest measure.

Solutions:

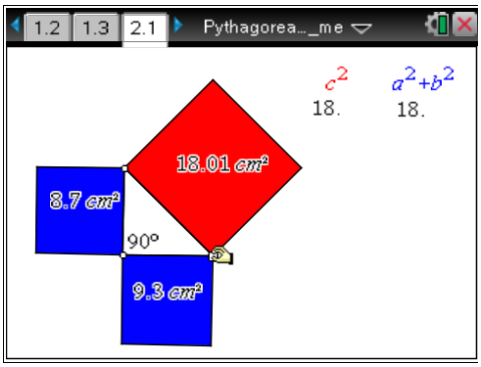
1. obtuse **2.** acute **3.** right **4.** obtuse **5.** Obtuse

TI-Nspire Navigator Opportunity: *Quick Poll* and *Screen Capture*

See Note 3 at the end of this lesson.

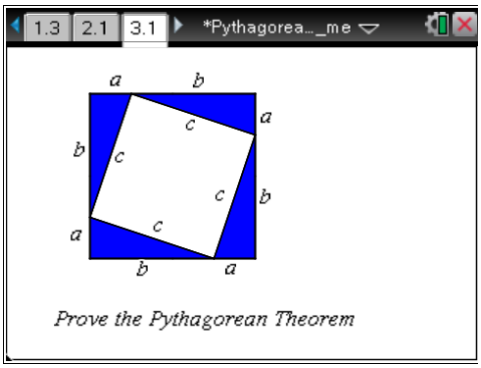
Problem 2 – Using squares

On page 2.1, a right triangle is drawn such that each side of the triangle is also a side of a square. Tell students to change the lengths of the sides of the triangle by dragging the vertices of the triangle. The triangle remains a right triangle and the sum of the areas of the two smaller squares always equals the area of the larger square. Have a class discussion on how this demonstrates the Pythagorean Theorem.



Problem 3 – Extension

Have students advance to page 3.1 and prove the Pythagorean Theorem by first substituting expressions into the equation $A_{\text{outer square}} = A_{\text{four triangles}} + A_{\text{center square}}$ and using algebra to simplify each side.



Solution:

$$A_{\text{outer square}} = A_{\text{four triangles}} + A_{\text{center square}}$$

$$(a+b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$$

$$a^2 + \cancel{2ab} + b^2 = \cancel{2ab} + c^2$$

$$a^2 + b^2 = c^2$$

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Quick Poll* and *Screen Capture*

Consider sending a *Quick Poll* asking what students observe when $c^2 < a^2 + b^2$. Also use Screen Capture to display a multitude of triangles showing this condition with the identical answer—acute.

Note 2
Problem 1, *Quick Poll, Live Presenter*

Consider sending a *Quick Poll* asking what students observe when $c^2 > a^2 + b^2$. Also use Screen Capture to display a multitude of triangles showing this condition with the identical answer—obtuse.

Note 3
Problem 1, *Quick Poll, Live Presenter*

Consider sending a *Quick Poll* asking students for the answers to Questions 1–5 on the Student Worksheet.