The Classic Box Problem Exploration

TI-84 PLUS CE FAMILY

Math Objectives

- Students will use multiple-linked, geometric (2D and 3D), and numeric representations to model a classic optimization problem.
- Students will make sense of problems and persevere in solving them.
- Students will model with mathematics. (CCSS Mathematical Practice).

Vocabulary

- optimization
- volume
- maximum
- rate of change

About the Lesson

- This lesson takes the classic optimization box problem and uses multiple mathematical representations to maximize the volume of the box.
- As a result, students will:
  - Create an algebraic model from geometric parameters.
  - Create a volume function from the algebraic model.
  - Find the maximum volume of a box based on the length of the side of the squares cut from each corner of a given piece of cardboard.

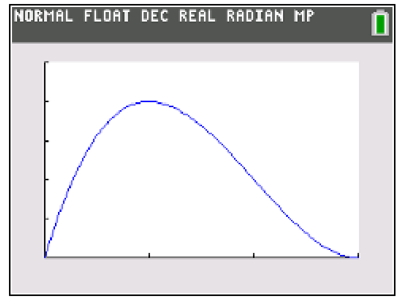
Teacher Preparation and Notes.

This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

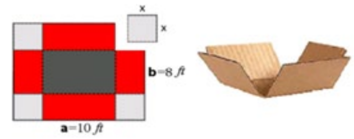
Lesson Files:

Student Activity

The_Classic_Box_Problem_Exploration_84CE_Student.pdf

The_Classic_Box_Problem_Exploration_84CE_Student.doc

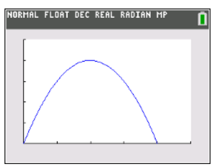
In this activity, you will create an open-box by taking an 8 ft by 10 ft sheet, cutting out square corners with sides of length x, and then bending up the sides. The goal of this activity is to figure out how to determine the size of the squares that result in the largest volume for the box.



Problem 1 – Creating the Box

1. Before we start the actual box problem, answer the following question. If you graphed the volume versus length of x, what shape do you think the graph will take? Explain your choice.

(a)



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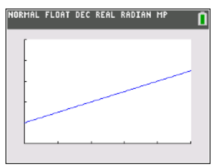

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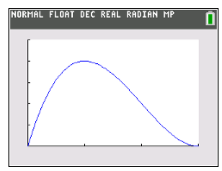

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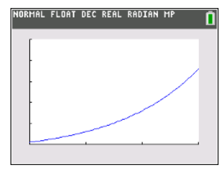


<u>Possible Solution:</u> The function that best models the volume is a cubic, which would be graph (c), but this question should be used as a way to get students talking about what the volume function looks like and why they believe it to look that way. Open discussion is key here.

2. What happens to the graph you chose in question 1 as the x value changes? Does the graph only increase? Does the graph increase and decrease? Explain.

<u>Possible Solution:</u> Because the length, x, is affecting both the length, width, and height of the box. Changing each of these will dramatically change the shape of the resulting box. You must have a realistic domain to create the function. An x to small makes the box very shallow and wide, while an x to big makes the box deep and narrow. You are trying to find that optimal volume. Before that optimal volume, the graph will increase and after the optimal volume, it will decrease.

Now, create a formula for volume using the 8x10 ft net. With respect to x...

3. What is the expression that represents width?

<u>Solution:</u> width = $(8-2x)$ or width = $(10-2x)$

4. What is the expression that represents length?

<u>Solution:</u> length = $(10-2x)$ or length = $(8-2x)$

5. What is the expression that represents height?

<u>Solution:</u> height = x

6. Put it all together. What is the function that represents the volume?

   Solution: $V(x) = (10 - 2x)(8 - 2x)(x)$ or $V(x) = 4x^3 - 36x^2 + 80x$

7. Check your function by graphing it on the handheld. Does this graph match the function you chose in question 1? Explain what you notice about the graph.

   Possible Solution: The point should follow along the function almost exactly, increasing before the maximum and decreasing after the maximum. Students could discuss how the rate of change of $V(x)$ is positive before the maximum and negative after the maximum.

Problem 2 – Optimization of the Box Problem

A square of side x-inches is cut out of each corner of a 10 in. by 14 in. piece of cardboard and the sides are folded up to form an open-topped box. $V(x)$ represents the volume of the box formed with respect to x.

8. Write the value of V as a function of x.

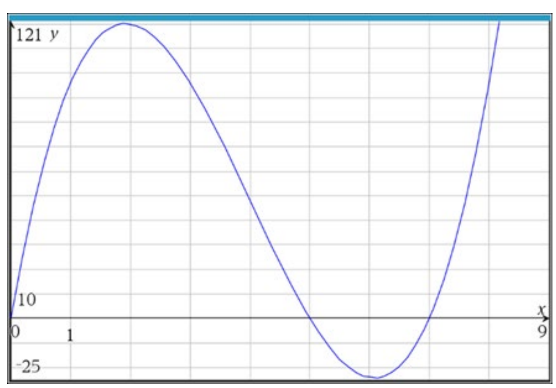
   Solution: $V(x) = x \cdot (10 - 2x) \cdot (14 - 2x)$ or $V(x) = 4x^3 - 48x^2 + 140x$

9. State the domain of the function $V(x)$.

   Solution: $0 < x < 5$ (This needs to be found using one half the smaller length of the cardboard. If it is larger than 5, one side of the box would not exist.)

10. Graph the function to find the maximum volume of the box. What is the maximum volume and what value of x gives the maximum volume?

    Solution: Maximum Volume: $120.164\ in.^3$
    The x-value that gives the maximum volume: $1.918\ in.$



11. How can you tell that this is the maximum value? Explain what is happening to the function, $V(x)$, before this maximum value and after the maximum value.

Solution: The function, $V(x)$, is increasing before the maximum, which means that the rate of change of the function is positive. The function is decreasing after the maximum, which means that the rate of change of the function is negative.