EXPLORATIONS ™

Activity 1

Objectives

- Recognize the slope-intercept form of a linear equation

- Investigate the effects of changes in *A* and *B* on a linear equation

- Use the guess-and-check strategy to manipulate the values of *A* and *B* to create a line of best fit on a scatter plot

How Many Drivers? Investigating the Slope-Intercept Form of a Line

Introduction

Any line can be expressed in the form *y = mx + b.* This form is called the *slope-intercept* form. In this activity, you will study the *slope-intercept* form of a line. When you have finished, you will be able to explain the effect of *m* and *b* on the graph of a line. You also will be able to fit a line to data by selecting values for *m* and *b* using the guess-and-check method.

The Effect of *m* and *b* on the Graph of a Line

1. You will begin this investigation by examining one instance of *y = mx + b*, when *m* = 3 and *b* = 5. On your graphing handheld, follow the steps below to enter the equation *y = 3x + 5.*

    a. Begin in **Function (Func)** mode and turn off any plots or equations previously turned on. (This screen shows both an unwanted equation and an unwanted plot.)
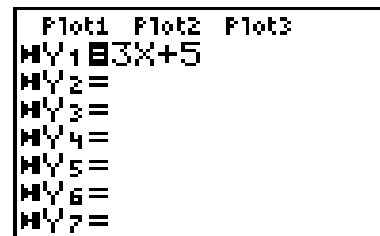
    b. To turn off a plot, press Y=, use the cursor keys (▲ ▼ ▶ ◀) to move to the highlighted plot, and press ENTER. To clear an equation, position the cursor anywhere in the right side of the equation, and then press CLEAR.

    c. Move the cursor to **Y1=**, and enter
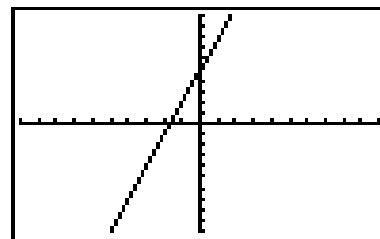3 $\boxed{\text{X,T,}\Theta,n}$ $\boxed{+}$ **5**.

```
Plot1 Plot2 Plot3
\Y1=3X+5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

2. Press $\boxed{\text{ZOOM}}$ **6** to select **6:ZStandard** and display the graph. The *x*-axis and *y*-axis will each have a range of ⁻10 to 10.

3. Enter and graph two more examples of *y* = *mx* + *b*.

    a. *y* = 3*x* + 2

    b. *y* = 5*x* + 2

What did you notice about the graphs of these two equations? How did the graphs of the line change for each equation? What effects do *m* or *b* have on the graph of the line? Why do you think this form, *y* = *mx* + *b*, is called the slope-intercept form?

You may not be ready to answer these questions yet. The short investigation that follows can give you a better understanding of the effect of *m* or *b* on the graph of a line. Before you begin, write your hypothesis about the effects of *m* and *b* on the graph of a line. You will check your hypothesis after the investigation.

Using the Transformation Graphing App to Investigate *m* and *b*

To investigate the effects of *m* and *b*, you will use a graphing handheld with the Transformation Graphing application. You must have the application installed and running on your graphing handheld to complete this activity.

To start the Transformation Graphing App:

1. Press APPS. Select **Transfrm**. (This may have a number or letter in front of it depending on the number of applications you have loaded.)

**2.** Press any key (except 2nd or ALPHA) to install the application.

If the application is already running, select **Continue**.

To graph the example:

1. Press Y=. Make sure the **Y=** editor is clear of all equations.

2. For **Y1**, enter **AX+B**. Press ALPHA A X,T,θ,*n* + ALPHA **B**.

*Note: You entered **Y=AX+B** in place of **Y=MX+B**, which is the form commonly found in textbooks, because the Transformation Graphing App uses only the coefficients A, B, C, and D.*
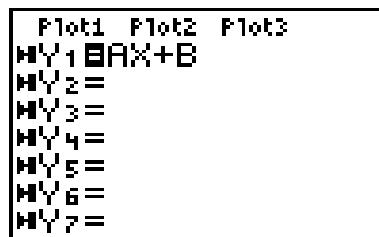
3. Press ⟨WINDOW⟩ ▲ to display the **SETTINGS** screen. If necessary, move the cursor until you highlight **>‖** and press ⟨ENTER⟩. The app is now set for Play-Pause mode.
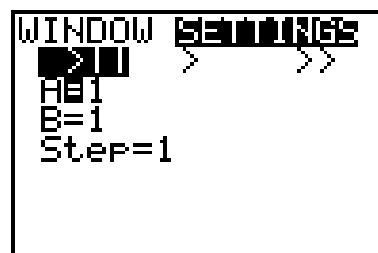
   Set the other **SETTINGS** as pictured. To make these selections, press ▼ 1 ▼ 1 ▼1. These settings define the starting values for the coefficients and the increment by which the coefficients will change.

4. Press ⟨ZOOM⟩ **6** to select **6:ZStandard** and display the graph. If **A=** is not highlighted (the equal sign will be highlighted), press ▼ until the **A=** is highlighted.
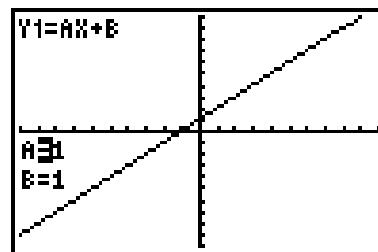
5. Press ▶ to increase or ◀ to decrease the value of *A*.

6. To enter values for *A* by hand, press the numeric keys for the value you want, and then press ⟨ENTER⟩. (Make sure that **A=** is still highlighted.) Give *A* both positive and negative values between ⁻10 and 10, including decimal values.

7. Use the same method to investigate the effect of *B* on the graph. Set **A=1**. To change the value of *B*, press ▼ to highlight **B=** and then press ▶ to increase or ◀ to decrease the value of *B*. You may also enter values for *B* by hand as you did for *A*.

Questions for Discussion

1. In a linear equation:

   a. As the value of *A* increases, what happens to the graph?

   b. As the value of *A* decreases, what happens to the graph?

   c. If the value of *A* is changed to a negative, how does the graph change?

2. In a linear equation:

   a. As the value of *B* increases, what happens to the graph?

   b. As the value of *B* decreases, what happens to the graph?

   c. If the value of *B* is changed to a negative, how does the graph change?

3. Refer to the hypothesis you wrote about the effects of *m* and *b* on the graph of the line. How accurate was your hypothesis?

4. Why do you think $y = mx + b$ is called the slope-intercept form of a line?

Problems

Use what you learned about the effects of *m* and *b* on the graph to solve these problems. Use your graphing handheld to verify your answers.

For each equation in items **1–7**, write the letter of its matching graph in the blank provided. Notice that the window range for both *x* and *y* is from ⁻10 to 10 with a scale of 1, which is the standard viewing window you selected in step **4**.
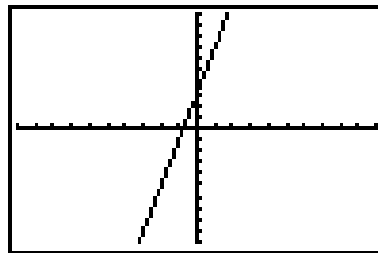
1. $y = 4x + 3$

_____

a.
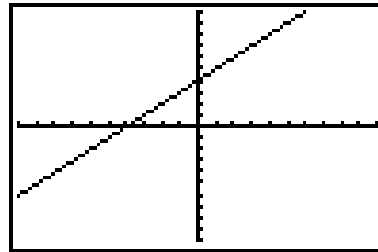
2. $y = 4x + 1$

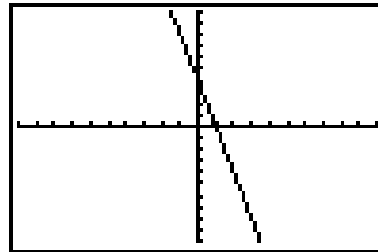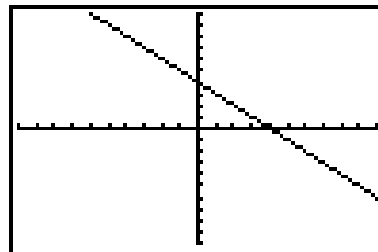_____

b.

3. $y = x + 4$

_____

c.

4. $y = x + 1$

_____

d.

5. $y = x - 4$

_____

e.

6. $y = {}^-x + 4$
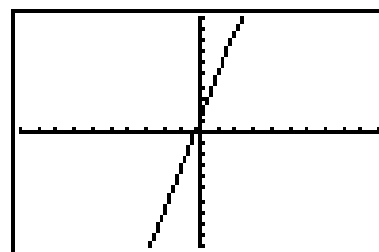
_____
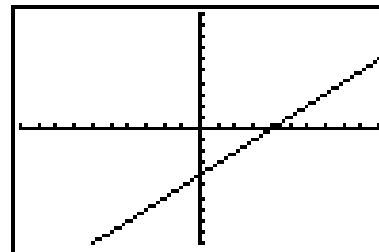
f.

7. $y = {}^-4x + 4$

_____
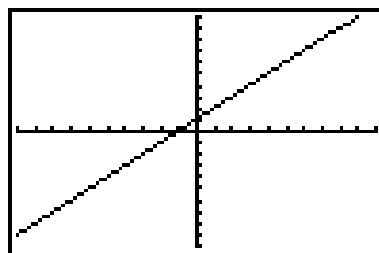
g.

8. Where does $y = 4x - 1$ cross the y-axis? How do you know?

9. Where does $y = {}^-3x + 2$ cross the y-axis? How do you know?

10. Which line is steeper, $y = 3x - 1$ or $y = x + 5$? How do you know?

11. Which two lines slant the same direction? How do you know?

$y = 2x + 5$

$y = 5x - 1$

$y = {}^-x + 3$

Use Your New Skill

In most states in the United States, an individual needs to be at least 16 years old to get a driver's license. This table shows the population from the 2000 census and the number of licensed drivers in selected states.

State	Population (in millions)	Licensed Drivers (in millions)
California	33.9	21.2
Florida	16.0	12.9
Georgia	8.2	5.6
Illinois	12.4	8.0
Michigan	9.9	6.9
Montana	0.9	0.7
New Mexico	1.8	1.2
New York	19.0	10.9
Pennsylvania	12.3	8.2
Texas	20.9	13.5

Source: Census 2000 data at http://quickfacts.census.gov/qfd/index.html
U.S. Department of Transportation, Federal Highway Administration
http://www.fhwa.dot.gov/ohim/hs00/dl22.htm

Create a scatter plot for these data to look for a relationship or pattern between the population and the number of licensed drivers.

Which set of data will be your independent variable, or *x*-values? _____

Which set of data will be your dependent variable, or *y*-values? _____

1. To enter the data in two stat lists, press [STAT] **1** to select **1:Edit**. Enter the values from the table into an empty list in the stat list editor. If no lists are empty, clear two lists. Press [▲] to highlight the list name and then press [CLEAR] [ENTER].

| L1	L2	L3	2
33.9	21.2	------	
16	12.9		
8.2	5.6		
12.4	8		
9.9	6.9		
.9	.7		
1.8	1.2		

L2(7) =1.2

2. Press [MODE], and verify that you are in **Func** mode. (**Func** is highlighted if you are in **Function** mode.)

3. Press [Y=] and clear any functions. Verify that all plots are off (not highlighted).

```
Plot1  Plot2  Plot3
\Y1=
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

4. Press [2nd] [STAT PLOT] **1** to select **1:Plot1** and press [ENTER]. Turn the plot on and enter the settings as shown at the right. (For **Xlist** and **Ylist**, be sure to use the list numbers in which your data were entered.)

```
Plot1  Plot2  Plot3
On  Off
Type: ▨ ⬚ ⬚
      ⬚ ⬚ ⬚
Xlist:L1
Ylist:L2
Mark: ▫ + ·
```

5. Press [ZOOM] **9** to select **9:ZoomStat** and display the plot using the data in the lists. Since some of the data in your lists exceeds the limits of the standard viewing window, you can let the handheld determine the window dimensions using the data in the lists. (For **Xlist** and **Ylist**, be sure to use the list numbers in which your data were entered.) What do you notice about the data? Is there a consistent trend in the relationship between each state's population and the number of licensed drivers in that state?
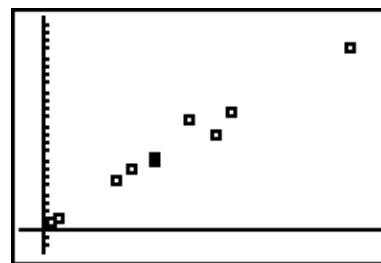
6. Give an estimated value for the slope and the *y*-intercept of a linear model to represent the data.
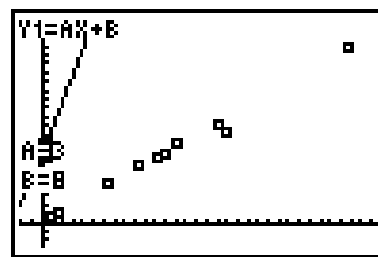
   Slope: _____

   y-intercept: _____

Using the Transformation Graphing App, you will find a line that represents an approximate fit of the data.

7. Press [Y=] to display the **Y=** editor. Enter **Y1=AX+B** for the equation. (Press [ALPHA] **A** [X,T,Θ,n] [+] [ALPHA] **B**.) Leave **Plot1** on.

   Press [◄] to move to the left of **Y1=**, and continue to press [ENTER] until the application is in Play-Pause mode (>‖).
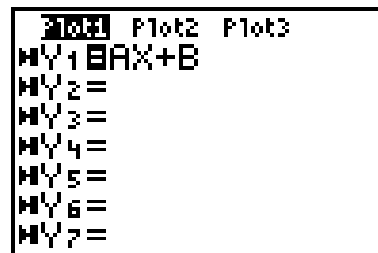
8. Because the window was set to display the plot, you do not need to change the **WINDOW** settings. Press [GRAPH] to display the plot with the app enabled. The line **Y=AX+B** is displayed with the most recent values of *A* and *B*. This line will probably have no relationship to your plot.

9. Enter your estimated values for the slope and *y*-intercept from step **6** as the values for *A* and *B*. To enter these values, use the up and down cursor keys ([▲] and [▼]) to highlight either **A=** or **B=**. Change the values of *A* and *B* using the cursor keys ([►] to increase values and [◄] to decrease values).
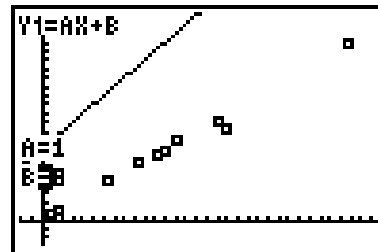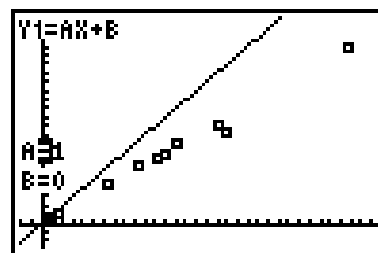
10. Adjust the values for *A* and *B* until you have a reasonable visual model with *A* and *B* values estimated to the hundredths place.

    What are your values for *A* and *B* in your model?

    A (slope): _____

    B (*y*-intercept): _____

Student Worksheet

Name _____

Date _____

How Many Drivers?

In most states in the United States, an individual needs to be at least 16 years old to get a driver's license. This table shows the population from the 2000 census and the number of licensed drivers in selected states.

State	Population (in millions)	Licensed Drivers (in millions)
California	33.9	21.2
Florida	16.0	12.9
Georgia	8.2	5.6
Illinois	12.4	8.0
Michigan	9.9	6.9
Montana	0.9	0.7
New Mexico	1.8	1.2
New York	19.0	10.9
Pennsylvania	12.3	8.2
Texas	20.9	13.5

Source: Census 2000 data at http://quickfacts.census.gov/qfd/index.html
U.S. Department of Transportation, Federal Highway Administration
http://www.fhwa.dot.gov/ohim/hs00/dl22.htm

In the guided activity, you estimated the value of *A* to be approximately 0.66 and the value of *B* to be about 0. That means the linear model is *y* = 0.66*x* + 0. When you made the scatter plot, you labeled the state population as the independent variable (*x*-values) and the licensed drivers as the dependent variable (*y*-values).

1. What is the unit of measure of the slope in this example?

_____

2. What do you think is the meaning of the slope of your model? What else could the slope represent?

_____

3. What does *B* represent in your model? Explain why 0 is a reasonable value for *B* in this example.

   _____

4. About how many licensed drivers does your model predict the state of Maryland, with a population of 5.3 million, should have?

   _____

   _____

   _____

5. Go to the Internet and find the population of your state. Use your model to predict the number of licensed drivers in your state.

   _____

Teacher Notes

Activity 1

How Many Drivers? Investigating the Slope-Intercept Form of a Line

This activity is an introduction to the slope-intercept form of a line. Its purpose is for students to recognize the effects of changes in the slope and the *y*-intercept on the graph of a line. In the guided activity, students will use the Transformation Graphing application to find an approximate linear model of actual data.

You should spend time talking about different models and what makes some better than others, but be careful that you do not look for a best model. If possible, present students with various models and have them discuss the reasonableness of each.

As a result of this activity, students are expected to be able to explain the effect of the parameters on the graph. They are not expected to graph a line by hand.

The activity will take approximately 50 minutes. It is appropriate for students in Algebra 1 and higher.

The activity includes only one modeling exercise. Students will be asked to do more modeling exercises in the next activity. If you would like to have students practice modeling more in this activity, you can use any linear data set from your textbook.

Make sure that students realize that only one equation can be graphed at a time while the Transformation Graphing App is installed and activated.

To deactivate the Transformation Graphing App:

1. Press [APPS]. Select **Transfrm**.

2. Select **1:Uninstall**.

```
TRANSFRM APP
1:Uninstall
2:Continue
```

Answers

Questions for Discussion

1. The value of *A* affects the slope (slant) of the line. The larger the value of *A* is, the steeper the line. When *A* is positive, the line slopes upward from left to right. When *A* is negative, the line slopes downward from left to right. Both *A* and ⁻*A* have the same steepness, but different directions. When *A* is zero, the line runs parallel to the *x*-axis.

2. The value of *B* affects where the line crosses the *y*-axis. The larger the value of *B*, the higher the line appears on the graph. When *B* is positive, the line intersects the *y*-axis in the upper quadrants (I and II). When *B* is negative, the line intersects the *y*-axis in the lower quadrants (III and IV). If *B* is zero, the line intersects the *y*-axis at the origin.

3. Answers will vary. Have students explain their hypotheses.

4. *A* indicates the slope of the line and *B* indicates the *y*-intercept.

Problems

1. b
2. f
3. a
4. e
5. g
6. d
7. c
8. At (0, ⁻1) because $B = ⁻1$.
9. At (0, 2) because $B = 2$.
10. The linear equation, $y = 3x - 1$, has a steeper slope because $A = 3$. Slope $= \frac{rise}{run}$, so a slope of **3** $= \frac{rise\ of\ 3}{run\ of\ 1}$.
11. $y = 2x + 5$ and $y = 5x - 1$ because the values of *A* in these two equations are positive. The value of *A* in the third equation is negative.

Use Your New Skill

x-values: population

y-values: licensed drivers

6. Answers will vary. Have students justify or explain their estimations.

10. Reasonable values for *A* and *B* will be approximately *A* = 0.66 and *B* = 0.

Student Worksheet

1. The unit of measure of the slope would be $\frac{\text{drivers}}{\text{population}}$.

2. The change in millions in the number of drivers per every 1 million change in the population; this is the percentage of licensed drivers in the total population.

3. Student responses should indicate that *B* represents the number of licensed drivers there would be if the population of the state (*x*) were zero. It makes sense that there would be no licensed drivers if there were no people living in the state.

4. Approximately 3.5 million

5. Answers will vary.