Average Roller Coaster

ID: 12215

Time Required
15 minutes

Activity Overview

In this activity, students will graphically discover how to find the average value of a function. This exploration uses animation and graphical observations to develop understanding. Self-check questions engage students and help deepen understanding. Multiple-choice exam-like questions are also included to apply what has been learned. Additionally, students will review concepts like average rate of change and using the Trapezoid Rule to find the definite integral when given only data.

Topic: Average Value of a Function

- *Average value of a velocity function*
- *Average rate of change versus average value of a function*
- *Relationship between velocity-time graph and displacement.*

Teacher Preparation and Notes

- *This activity can serve as an exploration and discovery of the average value of a function in the context of the speed of a roller coaster. The concept is shown to be generally applicable by using f(x) instead of v(t). Confusion between the average rate of change and average value is pointed out in the extension/homework exam-like problems. An understanding of the relationship between the definite integral of the velocity and displacement is demonstrated on page 1.10 of the TI-Nspire document. See Xtreme Calculus: Part 2 (activity #11488) for additional position, velocity, acceleration connections.*
- *Students will write their responses directly into the TI-Nspire handheld and/or on the accompanying handout. On self-check questions, after answering the question, students can press (menu) and select **Check Answer** (or press (ctrl) +▲).*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and teacher TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "12215" in the quick search box.***

Associated Materials

- *AverageRollerCoaster_Student.doc*
- *AverageRollerCoaster.tns*
- *AverageRollerCoaster_Soln.tns*

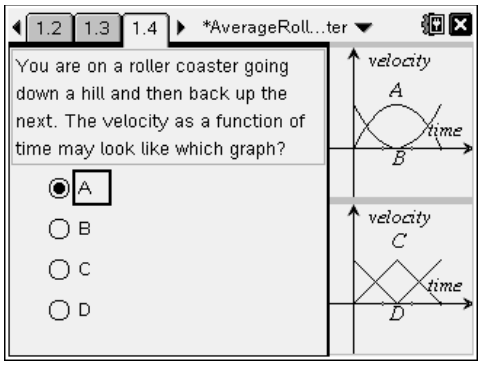
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

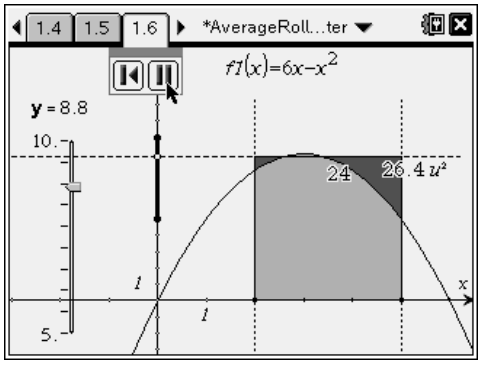
- *Average Value (TI-89 Titanium) — 3275*
- *Xtreme Calculus: Part 2 (TI-Nspire CAS technology) — 11488*

Part 1 – Discover/Explore Average Value of a Function

This activity begins with trying to help students make connections with what they already know about finding the average and how this needs to be improved to correctly find the average value of a function. A graph of the definite integral of a function is compared with the area that has the average value as its height and the difference of *b* and *a* as the base.



A common misconception is thinking that the graph is a picture of what happened rather than a mathematical representation. On multiple choice question 3, student commonly pick B instead of A.



For further discussion, or to help the students make additional observations to answer Question 6 on the student handout, ask the students to calculate or use the **Graph Trace** tool to determine the *f*(2) and *f*(5). What is the average of those values? How does this compare with the actual average value of the function?

The section concludes with graphical observations of the comparison of displacement and the velocity-time graph.

Student Solutions

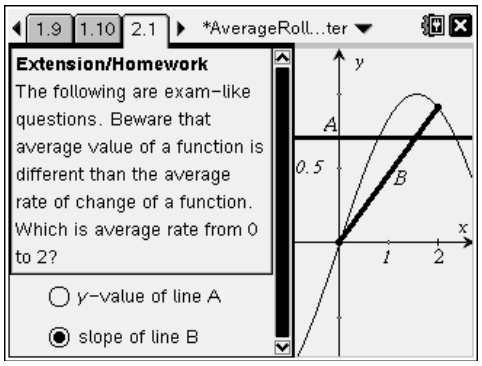
1. Add up the values and divide by the number of values to find the average of discrete values. $\frac{0+5+6+9+15}{5}=\frac{35}{5}=7$
2. No, 7 would be too small if the object traveled 15 m/s for four-fifths of the time.
3. Solution A. It starts out slow and speeds up then slows back down again.
4. The integral is 24. $\frac{24}{5-2}=\frac{24}{3}=8$
5. $\frac{30\frac{2}{3}}{5-1}=7\frac{2}{3}$
6. The area of the rectangle with a height equal to the average velocity is equal to the integral on that same interval. The average velocity is not equal to the average of the initial and final velocity.
7. The triangle represents the roller coaster. The diamond would be a very boring roller coaster idea. Observation: Another object, starting at the same time and place, but moving with a constant velocity equal to the average velocity would finish at the same time and place as the first object.

TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 1 at the end of this lesson.

Part 2 – Extension/Homework

This section enables students to try exam-like problems. Average rate of change and average value of a function are contrasted.



Many of the questions have the average rate of change or the average of two values for the wrong answers of the multiple-choice since these are common errors.

There are "calculator permitted" and "non-calculator questions." The final question has students use the data on a spreadsheet to find the average value with the Trapezoid Rule. The data is graphically shown to confirm the calculation on the last page.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Student Solutions

1. $\ln\left(e^{x^3}\right) = x^3$. So $\dfrac{\int_0^4 x^3 dx}{4-0} = \dfrac{1}{4}\left(\dfrac{1}{4}x^4\Big|_0^4\right) = 16$

2. $\dfrac{\int_1^e \dfrac{1}{x}dx}{e-1} = \dfrac{\ln e - \ln 1}{e-1} = \dfrac{1-0}{e-1} = \dfrac{1}{e-1}$

3. $\dfrac{f(4)-f(-2)}{4-(-2)} = \dfrac{0-|-8-4|}{6} = -2$

4. $\dfrac{\int_{-2}^4 \left|4x-x^2\right|dx}{4-(-2)} = \dfrac{1}{6}\left[\int_{-2}^0 \left(x^2-4x\right)dx + \int_0^4 \left(4x-x^2\right)dx\right] = 3.556$

5. $\frac{\int_1^b x^2 dx}{b-1} = \frac{13}{3}$. So $b = 3$ or -4. Students should use the Solve feature to solve for b.
This can be solved without the use of technology if students recall how to factor $b^3 - 1$.

$$\frac{1}{3}\frac{b^3-1}{b-1} = \frac{13}{3} \Rightarrow \frac{(b-1)(b^2+b+1)}{(b-1)} = 13 \Rightarrow$$

$$b^2 + b + 1 = 13 \Rightarrow b^2 + b - 12 = 0 \Rightarrow (b-3)(b+4) = 0$$

6. Students can use the spreadsheet by entering **=$\frac{1}{2}$(b1+b2)** in cell c1 and filling this down to c10. Then in c12 enter **=sum(c1:c10)** to find the approximate definite integral. Divide by (10 – 0) to find the average value. Or students can use the Trapezoid Rule formula

$$\frac{\frac{1}{2}(b_1 + 2\cdot b_2 + 2\cdot b_3 + 2\cdot b_4 + 2\cdot b_5 + 2\cdot b_6 + 2\cdot b_7 + 2\cdot b_8 + 2\cdot b_9 + 2\cdot b_{10} + b_{11})}{(b-a)}$$

Answer: 4.63 mi/min

TI-Nspire Navigator Opportunities

Note 1
Part 1, *Screen Capture*
This would be a good place to do a screen capture to verify students are answering the questions correctly and able to run the animation.

Note 2
Part 2, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.