Transformations - Dilation

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Student Activity

7 8 **9** 10 11 12

TI-Nspire™ | Investigation | Student | 50 min

Teacher Notes:

This activity uses the dynamic geometry functionality within the Geometry Application. Having dynamic functionality encourages students to explore, hypothesise and test.

The concepts covered here helps prepare students for higher level mathematics such as "invariance of properties under transformation, and the relationship between the determinant of a transformation matrix and the effect of the linear transformation on the area of a bounded region of the plane."
[VCE Specialist Mathematics]

Australian Curriculum Standards

AC9M7SP03
Describe transformations of a set of points using coordinates in the Cartesian plane, translations and reflections on an axis, and rotations about a given point

AC9M9SP02
Apply the enlargement transformation to shapes and objects using dynamic geometry software as appropriate; identify and explain aspects that remain the same and those that change

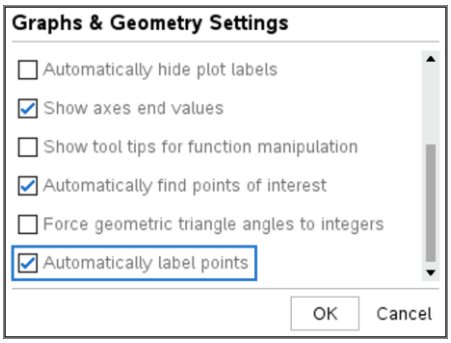
Calculator Instructions: Transformations

Create a new TI-Nspire document and insert a Geometry application.

In this activity it is useful to automatically label points.

menu > Settings

Make sure "Automatically label points" is checked.

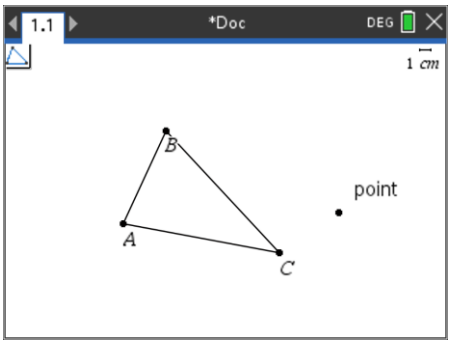


It's now time to draw a triangle.

menu > **Shapes** > **Triangle**

The triangle tool is active, indicated by the icon in the top left corner of the screen. A triangle can be created by clicking three different locations representative of the vertices of the triangle.

The vertices will be labelled A, B and C. ΔABC



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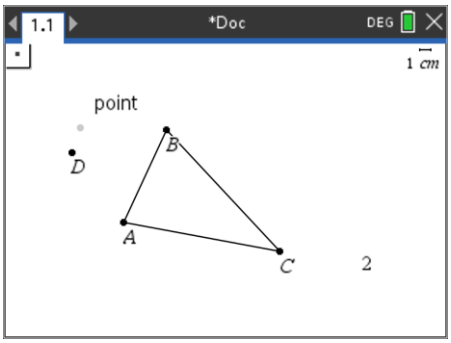
The next step is to insert some text. The text can be used to enlarge or reduce the size of the triangle.



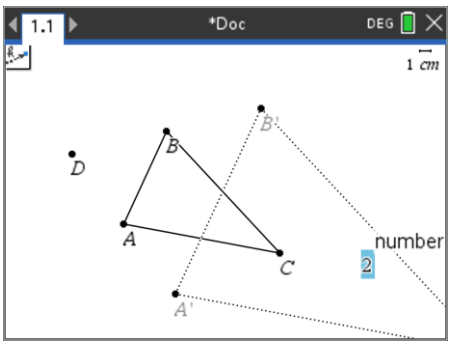
ctrl + menu > **Text**

Place the cursor away from the triangle and enter the number: **2**.

Place a point to the left of the triangle by pressing:

P … click to the left of the triangle to place a point.

The necessary tools have been put in place to dilate (enlarge or reduce) the size of the triangle.



menu > **Transformation** > **Dilation**

Click on point D, followed by the triangle and the dilation factor: 2.

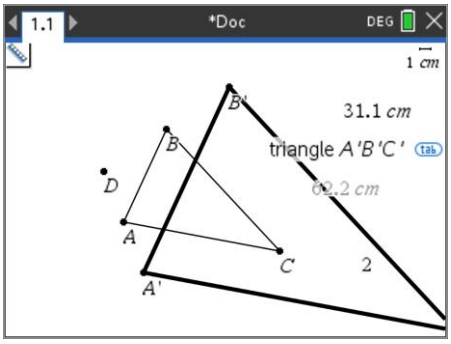
In the image shown here, triangle ABC has been dilated by a factor of 2 from point D. Press esc to release the dilation tool.

The transformed shape has vertices: A′B′C′.

Things to explore and contemplate:

- Grab and move point D to see how this impacts the transformed triangle.
- Edit the scale factor (2) to a number between 0 and 1.
- What happens if the scale factor is negative?
- Drag and move the vertices on the original triangle.
- Why can't the vertices on the transformed triangle be moved by grabbing?

Compare the perimeter of the original triangle and the transformed (dilated) triangles:



menu > **Measurement** > **Length**

Click on the original triangle, the perimeter of the triangle will be measured. To measure an individual side length, click on the corresponding vertices.

Repeat the above process and measure the area of the original triangle and the area of the transformed triangle.

Question: 1.

When the original triangle is dilated by a factor of 2, what happens to:

a) The perimeter of the triangle?

b) The area of the triangle?

Answer:

a) The perimeter of the triangle is doubled.

b) The area of the triangle is quadrupled (increased by a factor of 4).

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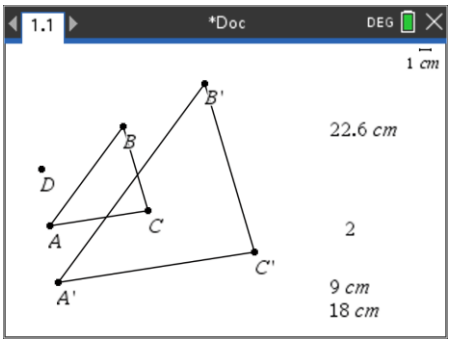
Compare each side length of the original triangle to the corresponding side of the transformed (dilated) triangle:

menu > **Measurement** > **Length**

Click on vertex A (original triangle) followed by vertex B (original triangle). The length of side AB will be measured.

Repeat this process for vertices A′ and B′ .



Question: 2.

When the original triangle is dilated by a factor of 2, what happens to ratio between corresponding side lengths of the original triangle compared with the dilated triangle?

Answer:

The side lengths are in the ratio: 1:2, that is, the side lengths are double.

Question: 3.

The measurement tool can also be used to measure the size of an angle. Measure corresponding angles in the original triangle and the dilated triangle. What happens to the angles in a dilation?

Answer:

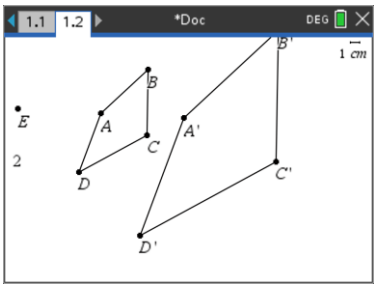
The angles remain the same. That is: ∠ABC = ∠A′B′C′

Question: 4.

Insert another Geometry application and draw a 4-sided polygon. Measure the perimeter and the area of each shape.

c) Compare the perimeter of the two shapes.

d) The area of the triangle?



Answer:

c) The perimeter of the quadrilateral is doubled.

d) The area of the quadrilateral is quadrupled (increased by a factor of 4).

Extension – Dilations on the Cartesian Plane

Insert a new problem into your document and add a Graphs application.

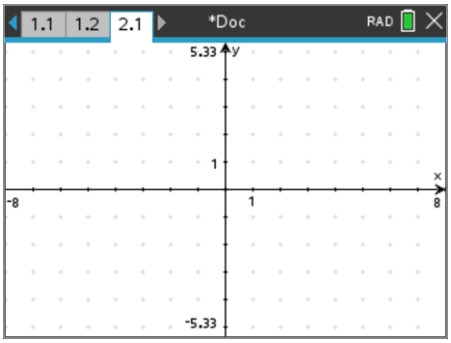
doc ▾ > **Insert** > **Problem**

Add a **Graphs** application into this problem.

A grid has been added to the background:

menu > **View** > **Grid** > **Dot Grid**

Match the window settings displayed opposite.



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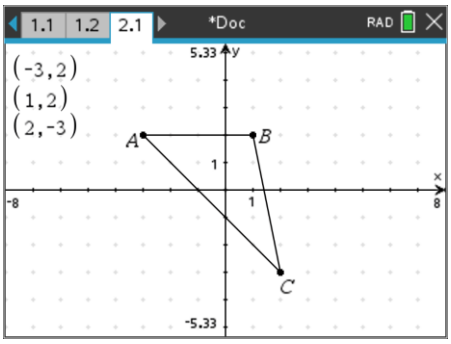
The Graphs application is also referred to as the Graphs & Geometry application because it also has most of the geometry features.

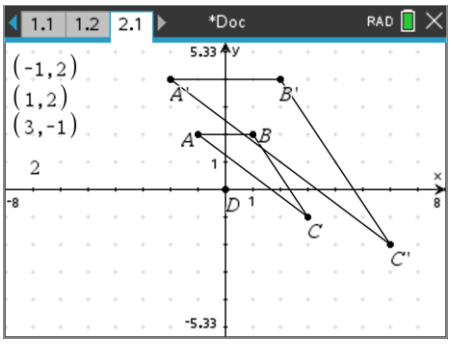
Use the Geometry options in the menu to draw a triangle, make sure the vertices of the triangle land on the grid.

The coordinates of each of the points A, B & C have also been determined. Hover the mouse over a vertex and press:

ctrl + menu > **Coordinates & Equations**



The dilation tool has been used to dilate the original triangle by a factor of 2 from the origin.



Question: 5.

With the original triangle coordinates at: A(-1, 2), B(1, 2) and C(3, -1), determine the coordinates of the points: A′, B′ & C′ when the dilation factor is set to 2.

Answer:

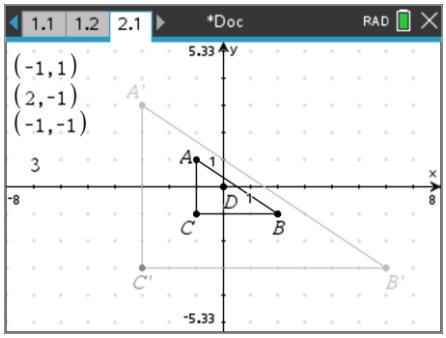
A′(-2, 4); B′(2, 4); C′(6, -2)

Question: 6.

The original triangle now has coordinates: A(-1, 1), B(2, -1) and C(-1, -1), determine the coordinates of the points: A′, B′ & C′ when the dilation factor is set to 3.

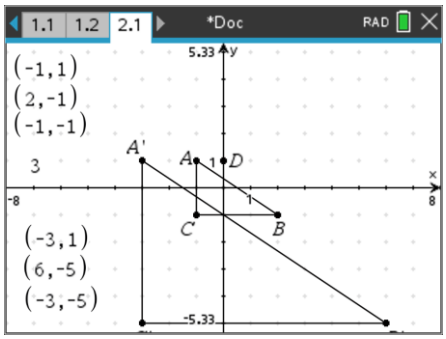
Answer:

A′(-3, 3); B′(6, -3); C′(-3, -3)



Question: 7.

The original triangle still has coordinates: A(-1, 1), B(2, -1) and C(-1, -1), however, point D has moved away from the origin.
The coordinates of the points: A′, B′ & C′ are shown when the dilation factor is set to 3.
Describe how moving point D has changed the outcome.



Answer:

The dilation factor is still the same so the individual lengths and perimeter will still be 3 times their original values, however the coordinates will change.
D has been moved one unit in the positive Y direction, the dilation is 'away from' D, so the points have moved one unit in the negative Y direction. Note that this is in the opposite direction of the movement of D.

Question: 8.

Using the same original triangle and dilation factor as Question 7, point D is moved to (3, 0).
Determine the coordinates of A′, B′ & C′ under this transformation.

Answer:

A′(-5, 2); B′(1, -2); C′(-5, -2)

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