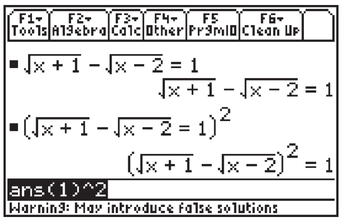
Learning Pre-calculus with the TI-89

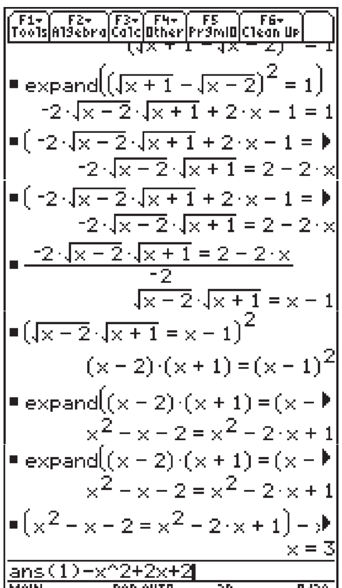
By John F. Mahoney, Editor

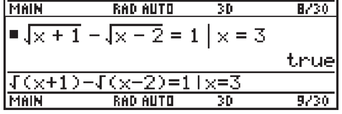
Pre-calculus is essentially a conglomeration of topics that bridge the gap between Algebra 2 and Calculus. The TI-89 can be used to help students learn and explore these topics.

Solving equations with radicals

Consider the equation: $\sqrt{x+1} - \sqrt{x-2} = 1$
which I displayed using the TI-89 ViewScreen in my Advanced Mathematics class. I asked my students what they would do first to solve this equation. Benesha suggested that we square both sides of the equation:



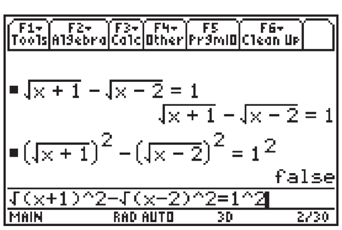




A) Note the warning "May introduce false solutions" - which appears when one squares both sides of an equation.
B) The class suggested we carry out the squaring of the left side by using the expand function.
C) We then added $1 - 2x$ to both sides.
D) Then divided both sides by -2
E) Now she suggested that we square both sides again, and again we also "multiplied out" the expressions using the expand function.
F) Finally, we added the expression $-x^2 + 2x + 2$ to both sides to solve for x.
G) To check our result we replaced x with 3 in the original equation and got the result we anticipated: *true*

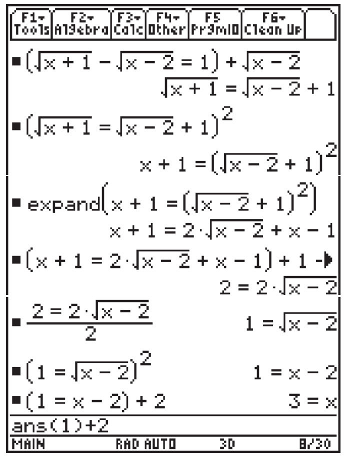
(Note that we were able to square both sides of the equation simply typing ^2.)

I asked the class if they could think of other methods to solve the equation. One student suggested that we square each part of the equation. I said, "Let's try it!"
Here's the result:



The calculator's response surprised me. It reached the conclusion not because the suggestion to square each term was wrong (which it certainly is), but because it would result in an expression: 3 = 1 , which is clearly never true.

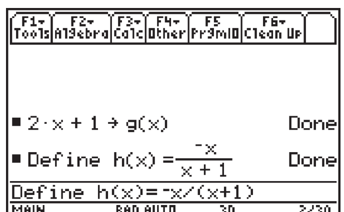
Rufus suggested another method. He suggested we add $\sqrt{x-2}$ to both sides before we squared both sides:

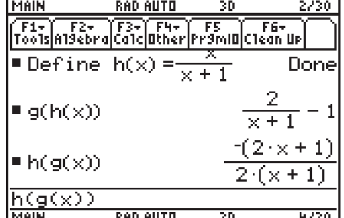


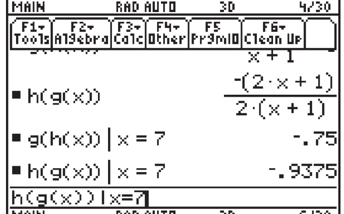
A) After we squared both sides, we multiplied out the right side by using the expand function.
B) We added the expression 1 - x to both sides
C) We then divided both sides by 2.
D) Then we squared both sides again.
E) Finally, we added 2 to both sides to get x = 3, as before.

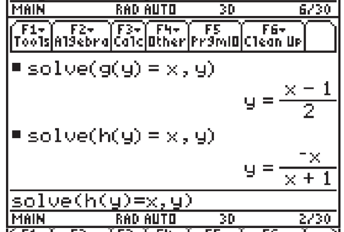
I then asked the class to try, on their own, to do a similar problem and to decide which method was probably quicker.

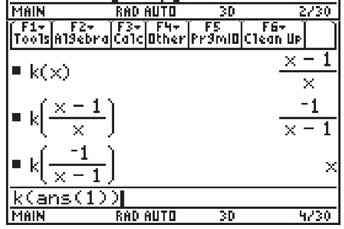
Composition of functions and inverses of functions











For most students, the study of composition of functions is the first occasion for them to see a process which is not commutative. Consider the functions:

$$g(x) = 2x + 1 \text{ and } h(x) = \frac{-x}{x+1}$$

We can use the TI-89 to explore the relations between them.

There are two ways to define a function.

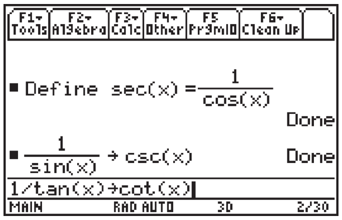
Here it is clear that $g(h(x)) \neq h(g(x))$ but just to be sure, we check the values of the two compositions when $x = 7$.

I tend to explain that one can find the inverse of a function by looking for a function, y ,which when you compose it with the original function, you get the identity function, x.
I use this notion in order to derive the inverses of our two functions.

Finally, the calculator can be used to help illustrate that, under composition, the function

$$k(x) = \frac{x-1}{x}$$

generates a group of order 3.

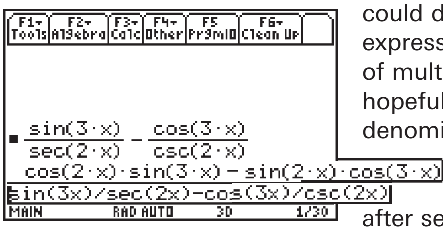


Trigonometric Identities

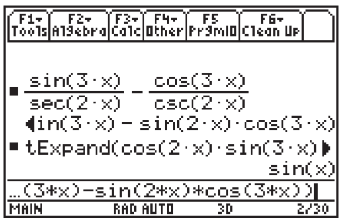
The TI-89 gives the user the opportunity to define functions. We can use it to define secant, cosecant, and cotangent.

A student who is asked to simplify

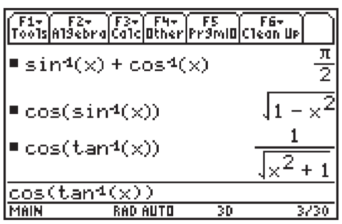
$$\frac{\sin 3x}{\sec 2x} - \frac{\cos 3x}{\csc 2x}$$



could decide to figure out expressions for trig functions of multiple angles, or hopefully, express the denominators in terms of sine and cosine the way the TI-89 does after secant and cosecant have been defined.

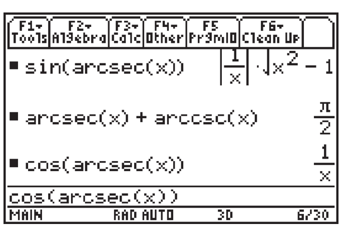


The calculator can use either tExpand or tCollect to simplify the expression using the identity for the sine of the difference of two numbers.

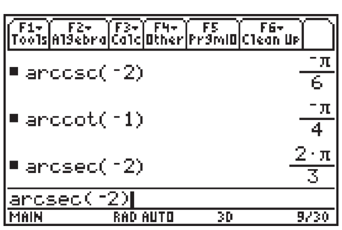


The identities for inverse trig functions are less well known by students. These identities are nonetheless important and are used in calculus.

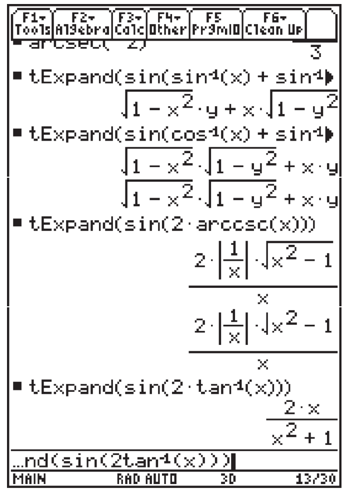
The other inverse functions can be defined in the traditional way.



Which enables us to examine other trig identities.



Unfortunately, mathematicians don't agree on the domain and ranges of the less common inverse trig functions. The definitions we used, above, produce ranges which may not be expected.



tExpand can be used with inverse functions, too.

By the way, historically[1], inverse functions were used in the calculation of π by using identities like:

$$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4},$$

$$2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}, \text{ and}$$

$$4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239} = \frac{\pi}{4}$$

along with the series:

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

Investigation: Ask your students to use the calculator to "tExpand" cos(nx) and sin(nx) for $2 < n < 10$ to see if they can identify patterns which would predict the identities for the next values of n. Note: There are indeed patterns which can be seen by first setting the real parts (and then the complex parts) of de Moivre's equation

$$(\cos(nx) + i\sin(nx)) = (\cos x + i\sin x)^n$$

equal to each other.

1 Pre-Calculus Mathematics, Merrill Shanks, et al, 1976, page 192

Note from the editors:

The extended "screen shots" appearing in this story are the result of cropping images with a word processor. For users of Microsoft's Word for Windows, first copy the calculator's screen to the Clipboard using the TI-Graph Link and then paste it into a document. Clicking on the image in the document allows you to resize it. But, if you hold the Shift key down while you click on it, then you'll be able to crop it. You can crop out the bottom of one screen shot and the top of another in order to give the appearance of extended screen shots. Obviously you can do equivalent cropping on the right and left in order to merge screen shots horizontally. This process works with other word processors and computers, too.