Implicit Differentiation

Name ________________________

Class ________________________

Problem 1 – Finding the Derivative of $x^2 + y^2 = 36$

The relation, $x^2 + y^2 = 36$, in its current form *implicitly* defines two functions, $f_1(x) = y$ and $f_2(x) = y$. Find these two functions by solving $x^2 + y^2 = 36$ for y.

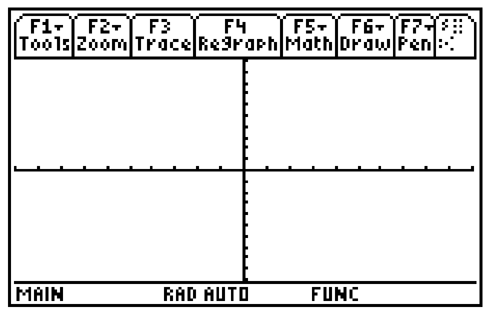
$f_1(x) =$ $\qquad\qquad$ $f_2(x) =$

Substitute the above functions in the original relation and then simplify.

$x^2 + (f_1(x))^2 = 36$ $\qquad\qquad$ $x^2 + (f_2(x))^2 = 36$

This confirms that $f_1(x)$ and $f_2(x)$ *explicitly* defines the relation $x^2 + y^2 = 36$.

Graph $f_1(x)$ and $f_2(x)$ on the same set of axis and then draw it in the space to the right. Imagine that you were asked to find the slope of the curve at $x = 2$.



- Why might this question be potentially difficult to answer?

- What strategies or methods could you use to answer this question?

One way to find the slope of a tangent drawn to the circle at any point (x, y) located on the curve is by taking the derivative of $f_1(x)$ and $f_2(x)$.

$\frac{dy}{dx} f_1(x) =$ $\qquad\qquad$ $\frac{dy}{dx} f_2(x) =$

Check that your derivatives are correct by using the **Derivative** command (press **F3:Calc >1:*d*(diffferentiate**) on the *Calculator* screen.

Substitute 2 for x to determine the slope of the tangents to $x^2 + y^2 = 36$ at $x = 2$.

$\frac{dy}{dx} f_1(2) =$ $\qquad\qquad$ $\frac{dy}{dx} f_2(2) =$

Another way to find the slope of a tangent is by finding the derivative of $x^2 + y^2 = 36$ using *implicit differentiation*. On the *Calculator* screen press **F3:Calc > D:impDif(** to access the **impDif** command. Enter **impDif(x² + y² = 36, x, y)** to find the derivative.

$$\frac{dy}{dx} =$$

Use this result to find the slope of the tangents to $x^2 + y^2 = 36$ at $x = 2$. First you will need to find the y-values when $x = 2$.

$\frac{dy}{dx}(2, y) =$ $\frac{dy}{dx}(2, y) =$

- Is your answer consistent with what was found earlier?

- Rewrite the implicit differentiation derivative in terms of x. Show that, for all values of x and y, the derivatives of $f_1(x)$ and $f_2(x)$ that you found earlier are equal to the result found using the **impDif** command.

Problem 2 – Finding the Derivative of $x^2 + y^2 = 36$ By Hand

To find the derivative of a relation $F(x, y)$, take the derivative of y with respect to x of each side of the relation. Looking at the original example, $x^2 + y^2 = 36$, we get:

$$\frac{d}{dx}\left(x^2 + y^2\right) = \frac{d}{dx}(36)$$
$$\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(y^2\right) = \frac{d}{dx}(36)$$

Evaluate the following and by hand.

$\frac{d}{dx}\left(x^2\right) =$ $\frac{d}{dx}(36) =$

Use the **Derivative** command to find $\frac{d}{dx}\left(y^2\right)$. Set up the expression up as $\frac{d}{dx}\left(y\left(x\right)^2\right)$. Notice that *y*(*x*) is used rather than just *y*. This is very important because it reminds the calculator that *y* is a function of *x*.

$$\frac{d}{dx}\left(y^2\right) =$$

You have now evaluated $\frac{d}{dx}\left(x^2\right)$, $\frac{d}{dx}\left(y^2\right)$, and $\frac{d}{dx}(36)$. Replace these expressions in the equation $\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(y^2\right) = \frac{d}{dx}(36)$ and solve for $\frac{dy}{dx}$.

Compare your result to the one obtained using the **impDif** command.

Problem 3 – Finding the Derivative of $y^2 + xy = 2$

The relation, $y^2 + xy = 2$, can also be solved as two functions, $f_1(x)$ and $f_2(x)$, which *explicitly* define it.

- What strategy can be used to solve $y^2 + xy = 2$ for *y*?

Solve $y^2 + xy = 2$ for *y* and use the **Solve** command (press **F2:Algebra > 1:solve(**) to check your answer.

The derivative of $y^2 + xy = 2$ can then be found by taking the derivatives of $f_1(x)$ and $f_2(x)$. However, the derivative can be found more easily using implicit differentiation.

Use implicit differentiation to find the derivative of $y^2 + xy = 2$. Check your result by using the **impDif** command. (*Hint*: The product rule must be used to find the derivative of *xy*.)

$$\frac{dy}{dx} =$$

Use the derivative you found for $y^2 + xy = 2$ to calculate the slope at $x = -6$. First you will need to find the *y*-values when $x = -6$.

$\frac{dy}{dx}\left(-6, y\right) =$ $\qquad\qquad$ $\frac{dy}{dx}\left(-6, y\right) =$

Verify your result graphically. Graph the two functions, $f_1(x)$ and $f_2(x)$. Then use the slopes and points to graph each tangent line.

Extension – Finding the Derivative of $x^3 + y^3 = 6xy$

The relation $x^3 + y^3 = 6xy$ cannot be solved explicitly for *y*. In this case implicit differentiation must be used.

- Find the derivative of $x^3 + y^3 = 6xy$ and use the **impDif** command to verify your result.

  $$\frac{dy}{dx} =$$

  Use this result to find the slope of the tangents to $x^3 + y^3 = 6xy$ at $x = 1$. (*Hint*: Use the **solve** command to find the *y* values that correspond to $x = 1$.)