EXPLORATIONS

Activity 4

Walking the Line

Teacher Notes

Concepts

- Slope
- Using data to make predictions
- Linear graphs and tables
- Linear equations and inequalities

Calculator Skills

- Calculating slope
- Using constant statements to generate values in a table
- Using variable expressions

Materials

- TI-30X IIS
- Student Activity pages (p. 38-42)
- Coordinate grid paper

Objective

- In this activity, students will use linear functions to model and solve problems in situations with slope and a constant rate of change.

Topics Covered

- Representing situations with variables in expressions, equations, and inequalities
- Using tables and graphs as tools to interpret expressions, equations, and inequalities
- Using the power of symbolic mathematics to solve real-world problems

Introduction

In addition to the invoice price, many new automobiles have information on the sticker giving the number of miles per gallon for city driving and the number of miles per gallon on the highway. A certain new car is advertised to get 17 miles per gallon in the city and 22 miles on the highway. The car has a full tank of gasoline (20 gallons), and is driven for 180 miles on the highway to a distant city. How many miles of city driving could you get with the gasoline remaining in the tank after the 180-mile highway trip? Determining rate of change and slope are helpful in solving problems like this one.

Investigation

1. Explain how to find the slope of the line passing through two points. Suppose a line passes through the points whose coordinates are P_1 (0, ⁻3) and P_2 (9, 9). Find the slope using the formula

   slope = m = $\frac{y_2 - y_1}{x_2 - x_1}$ where P_1 (x_1, y_1) and P_2 (x_2, y_2) are the coordinates of the two points.

2. Solve the problem on your overhead calculator, and change the decimal result to a fraction.

Press:	The calculator shows:
[(] 9 [−] [(−)] 3 [)] [÷] [(] 9 [−] 0 [)] [ENTER]	(9 − ⁻3) / (9 − 0→ 1.333333333 DEG
[2nd] [F⇔D] [ENTER]	Ans ▸ F ↔ D 1 ␣ 1/3 DEG
[2nd] [A b/c ↔ d/e] [ENTER]	Ans ▸ $A^b/_c \leftrightarrow {}^d/_e$ 4/3 DEG

3. Suppose a line passes through the points P_1 (0, ⁻4) and P_2 (5, 6). Use the recall feature of the TI-30X IIS to edit the original key sequence to calculate the slope.

Press:	The calculator shows:
[▲] [▲] [▲]	(9 − ⁻3) / (9 − 0→ DEG
[▶] 6 [▶] [▶] 4 [▶] [▶] [▶] 5 [ENTER]	(6 − ⁻4) / (5 − 0→ 2 DEG

4. Revisit the gasoline mileage problem in the Introduction. Have students work in pairs to find the solution to the problem. Give the expression and result.

   (20 − (180 ÷ 22)) /17 = .695

Wrap-Up

Student Activity Part 1 should be done individually. The problems in *Student Activity Part 2* and *Student Activity Part 3* make effective group work. Students should discuss which approach to take, and you should make sure that the approach is appropriate. Finally, students should share their conclusions to see if they have interpreted the results correctly. Remind students to use 2nd [FIX] 2 for the problems involving dollars and cents.

Extension

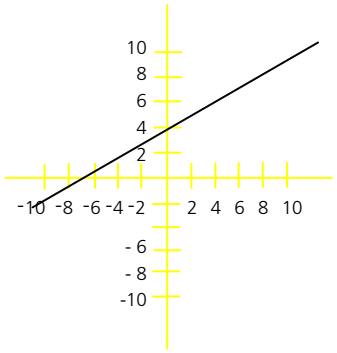
Have students work in groups of four. Give each group of students a graph of an ellipse with the foci clearly marked. Have the students choose eight points on the ellipse that are approximately equally spaced and measure the distance to each focus. Call the distances x and y, and build a table. Show that an equation of the form $x + y = k$ models the data.

Solutions Part 1

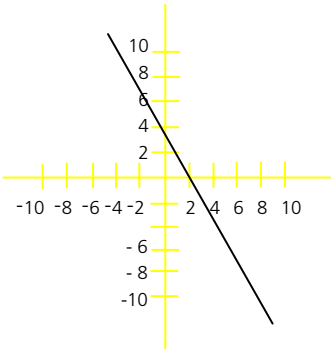
For each of the following ordered pairs of numbers:

- Plot the points on a coordinate grid and draw a line through the points.
- Find the slope of the line passing through the points.

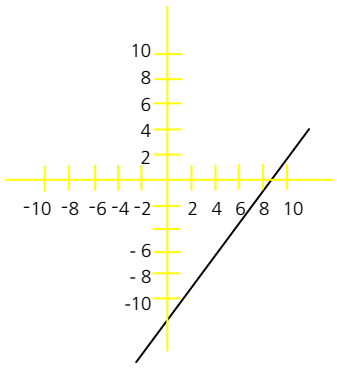
1. (-1, 3) and (5, 6) *Slope = 0.5*



2. (3, -2) and (-1, 5) *Slope = -1.75*

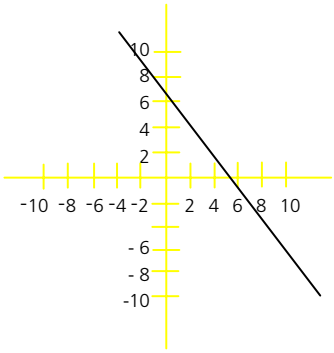


3. (4, -7) and (-1, -14)



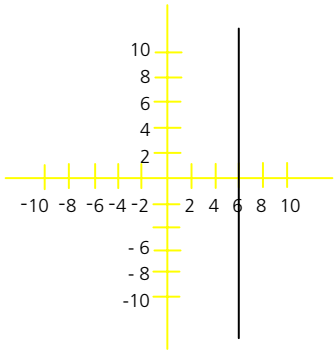
Slope = -1.4

4. (-2, 9) and (11, -7)



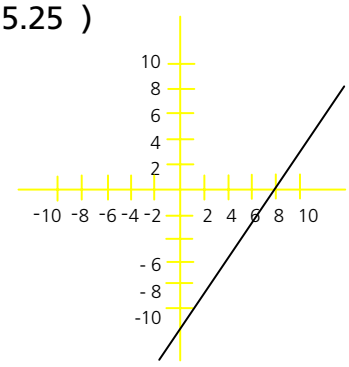
Slope = -1.231

5. (6, 15) and (6, -20)



It is a vertical line, so the slope is undefined (cannot divide by zero).

6. (3.5, -7.5) and (-2.75, 15.25)



Slope = -3.64

7. In 1975 a new Ford Mustang cost $4,960. In 1981, only six years later, a comparable new Ford Mustang cost $7,920. Calculate the slope of the line that would connect these two data points on a graph. What is the meaning of the slope in this problem situation?

   $\frac{\$7920 - \$4960}{1981 - 1975} = \$493.33.$

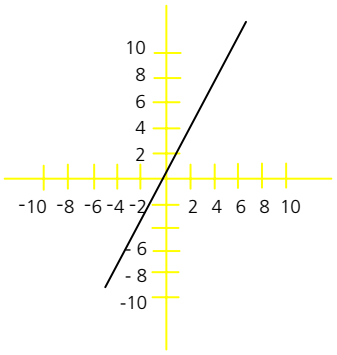
   This means that the price of the car increases, on average, $493.33 per year.

Solutions Part 2

The enlargement factor of film at a given distance from a screen is defined to be the ratio of the size of the projected image on the screen to the size of the object on the film. Complete the table of slide projector enlargement factors, and then use the data to complete the problems that follow.

Distance of projector from the screen	Size of object (film)	Size of object (screen)	Enlargement factor
1 yard	16 mm	32 mm	*2*
2 yards	16 mm	64 mm	*4*
3 yards	16 mm	96 mm	*6*
4 yards	16 mm	128 mm	*8*

1. Make a plot on coordinate grid paper of the pairs (*Distance*, *Enlargement Factor*) from the data above. Draw a line through the pairs.



   Use the model or the pattern in the table to estimate the enlargement factor when the projector is:

   - 3.5 yards from the screen *7*
   - 7.5 yards from the screen *15*

2. Describe the pattern of change in the enlargement factor as the projector is moved farther away from the screen.

   The enlargement factor is twice the distance from the screen.

   How is this pattern shown in the table and in the graph?

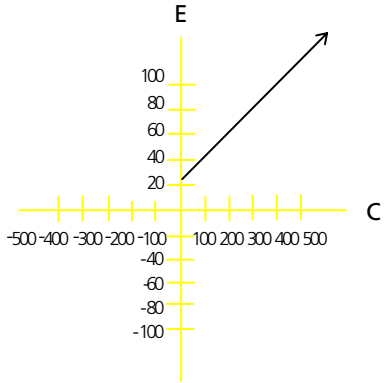
   In the tables, the enlargement factor increases by 2 as the distance increases by 1. In the graph, the pattern of change is shown by a vertical change of 2 units for each horizontal change of 1 unit.

Solutions Part 3

Joanne is a waitress at La Familia Restaurant. The management of the restaurant automatically adds a 15% tip on all customers' bills as a courtesy to their waitresses and waiters. Joanne gets paid $25.00 per shift plus tips for working the 5 p.m. to 11 p.m. shift at La Familia.

1. Write an equation to model Joanne's earnings (E) based on the total of her customers' orders (C). *E = 25.00 + .15C*. Use your calculator to complete the table, and then draw a graph on the coordinate grid.

Total of customers' orders (C)	Process	Value of Joanne's earnings (E)
0	$25.00 + 15%(0)	$25.00
100	*25.00 + 15%(100)*	*40.00*
200	*25.00 + 15%(200)*	*55.00*
300	*25.00 + 15%(300)*	*70.00*
400	*25.00 + 15%(400)*	*85.00*
500	*25.00 + 15%(500)*	*100.00*
600	*25.00 + 15%(600)*	*115.00*



2. If the customers' orders totaled $210, what calculation will give Joanne's wage for the evening?

   25.00 + 15% (210) = $56.50

3. If Joanne's earnings for last night were $57.00, write and solve an equation showing the total amount for her customers' orders.

   25.00 + .15C = 57.00 15C = 32.00 C = $213.33

4. What is the smallest amount of money that Joanne could make in one evening?

   $25.00

   How is her smallest amount reflected in the equation?

   If the total of customer orders (C) is 0.

5. After six months at La Familia, Joanne will receive a raise to $28 per shift. She will continue to receive 15% in tips. Write an equation to model her new earnings.

   E = 28.00 + .15C

6. What if, after six months at La Familia, Joanne has a choice of getting paid $28 per shift plus 15% in tips or staying at $25 per shift but within 16% tips. Which one should she choose? How would she decide?

   If Joanne has more that $300 in orders, she would make more by choosing $25.00 per shift with 16% tips. The more orders she has, the more money she earns.

28	15%	100	43
25	16%	100	41
28	15%	200	58
25	16%	200	57
28	15%	300	73
25	16%	300	73
28	15%	301	73.15
25	16%	301	73.16

Student Activity 4

Name _____

Date _____

Walking the Line

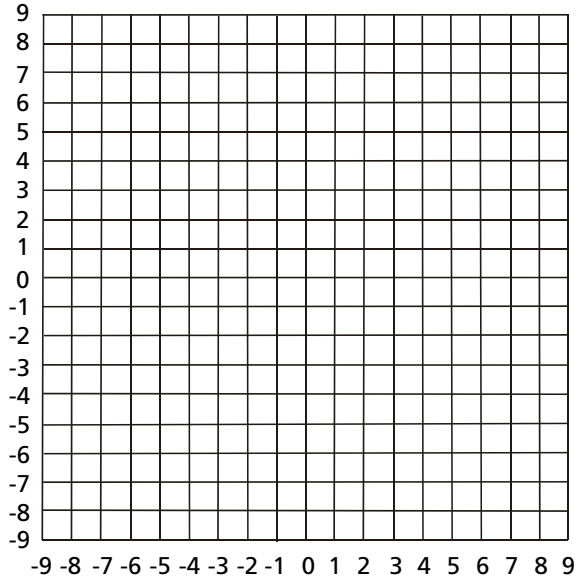
Objective: *You will learn how to find the slope of a line and how to use the slope to solve problems involving rate of change.*

Part 1: Graphing and Slope

For each of the following ordered pairs of numbers:

- Plot the points on a coordinate grid and draw a line through the points. (In # 3 through #6, you will need to determine the scale and add a label.)
- Find the slope of the line passing through the points.

1. (-1, 3) and (5, 6)



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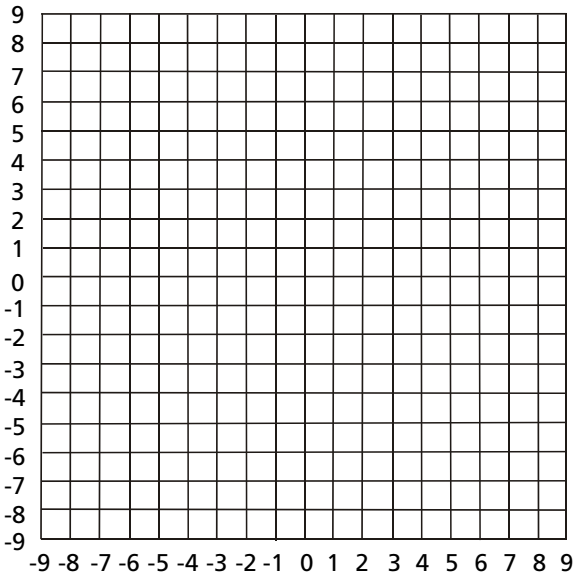

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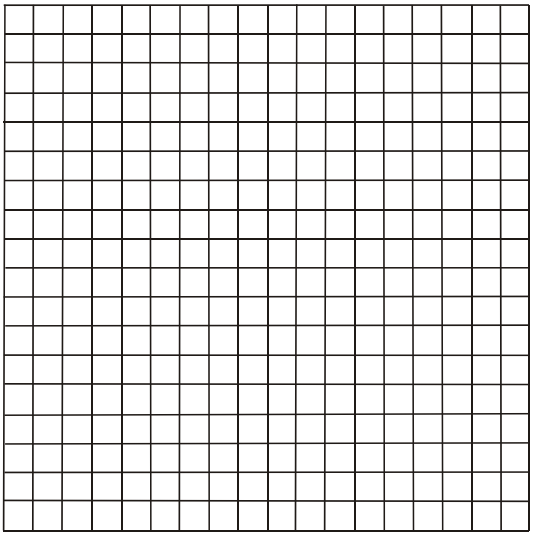

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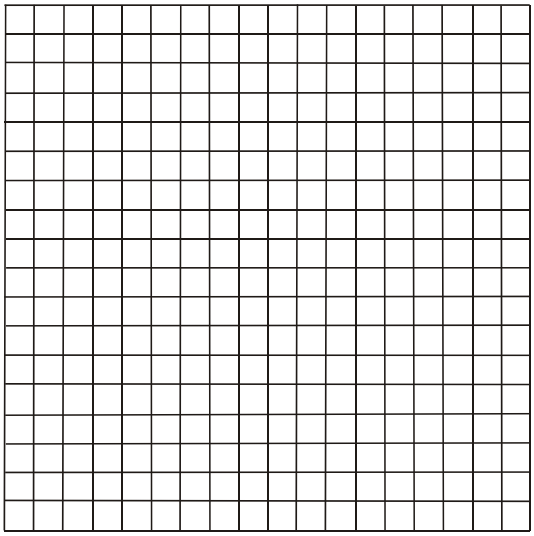

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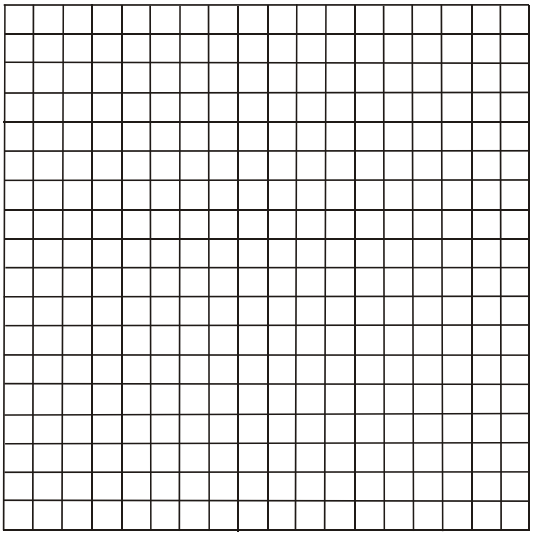

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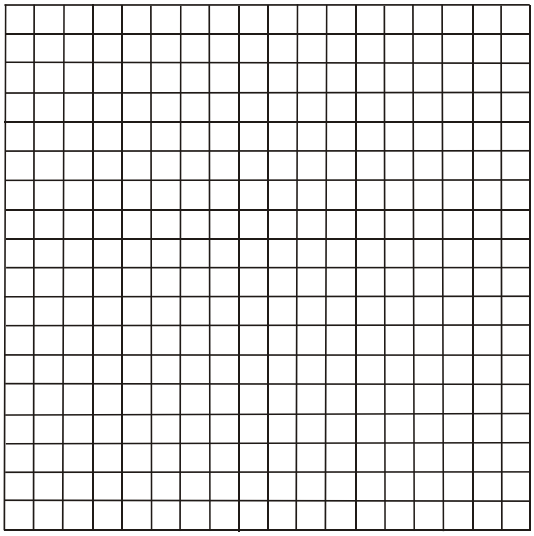

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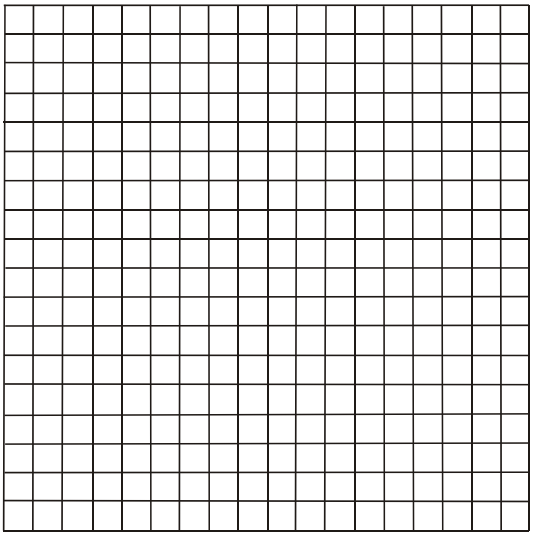

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   How is this pattern shown in the table and in the graph?

Part 3: Predicting with Linear Models

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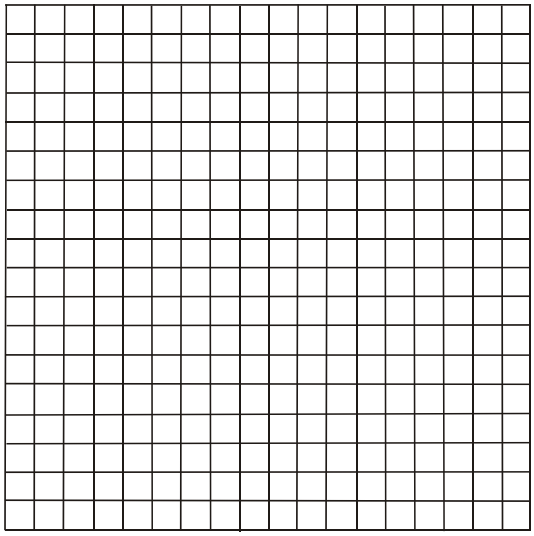

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